

Implementing Abstractions

Part Two

Previously, on CS106B...

```
class OurStack {
public:
    OurStack();

    void push(int value);
    int peek() const;
    int pop();

    int size() const;
    bool isEmpty() const;

private:
    int* elems;
    int allocatedSize;
    int logicalSize;
};
```

```
class OurStack {  
public:  
    OurStack();  
  
    void push(int value);  
    int  peek() const;  
    int  pop();  
  
    int  size() const;  
    bool isEmpty() const;  
  
private:  
    int* elems;  
    int  allocatedSize;  
    int  logicalSize;  
};
```

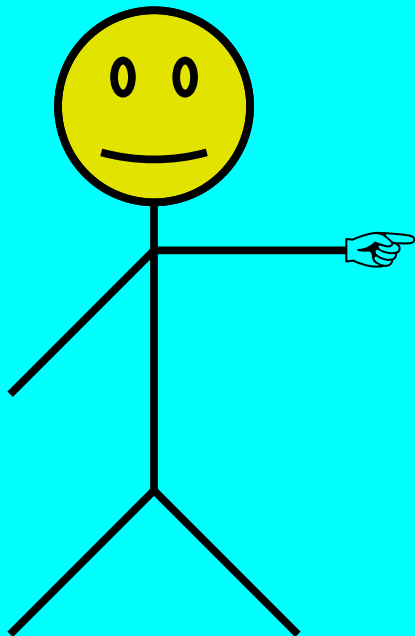
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class OurStack {
public:
    OurStack();

    void push(int value);
    int peek() const;
    int pop();

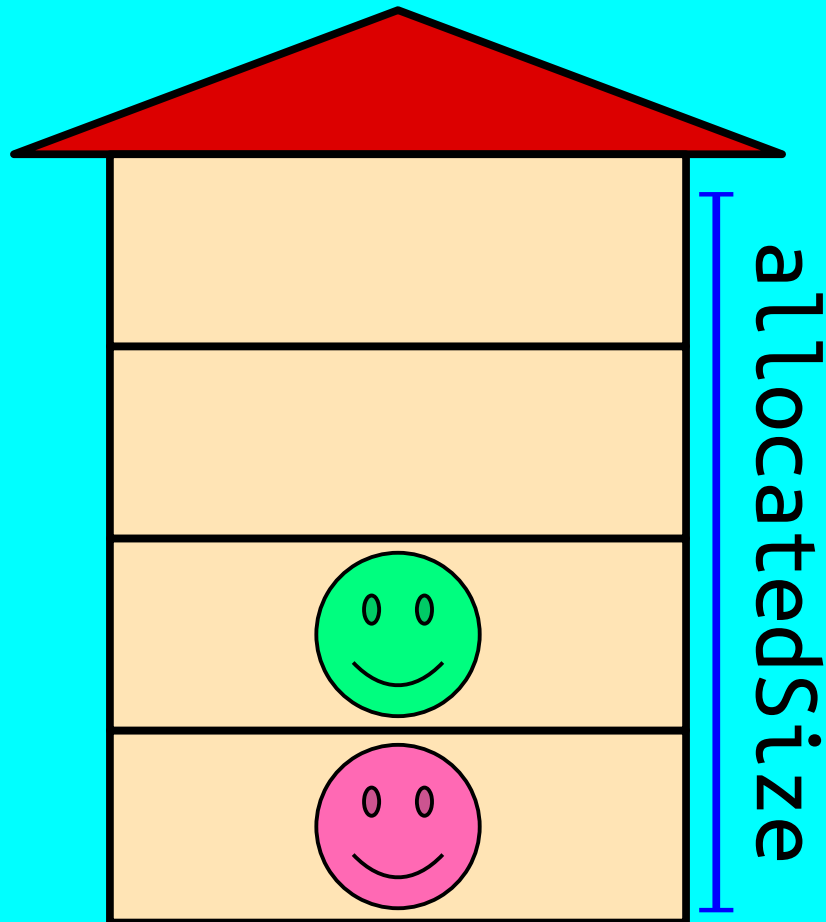
    int size() const;
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private:
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};
```

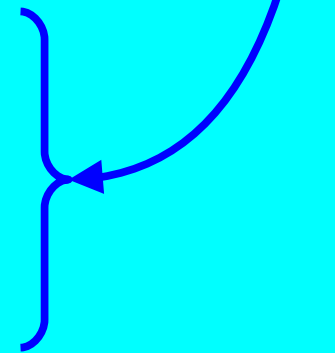
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private:  
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```



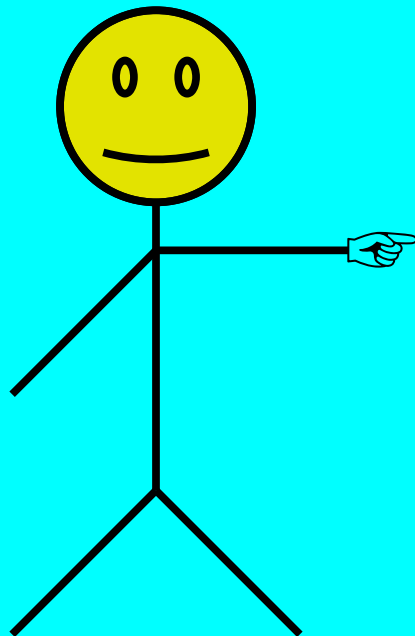
elems



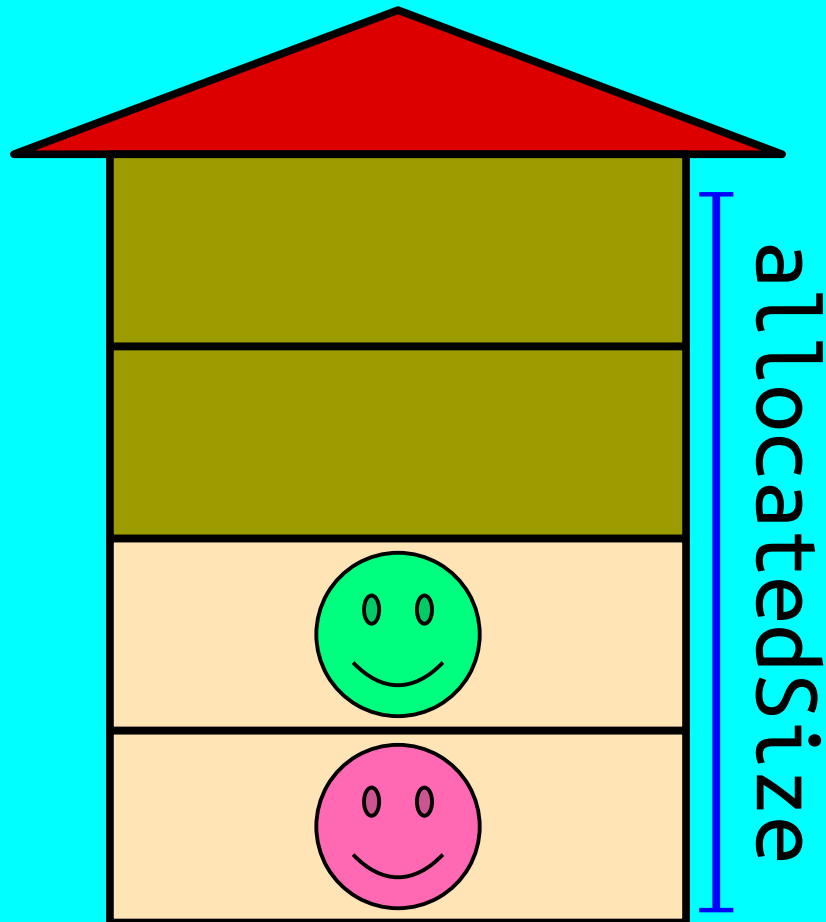
logicalSize



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private:  
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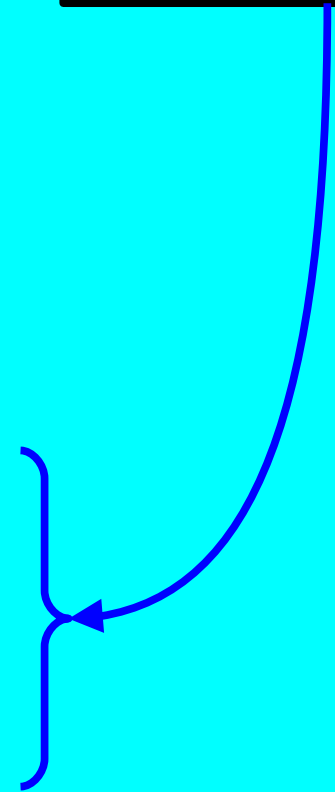


elems

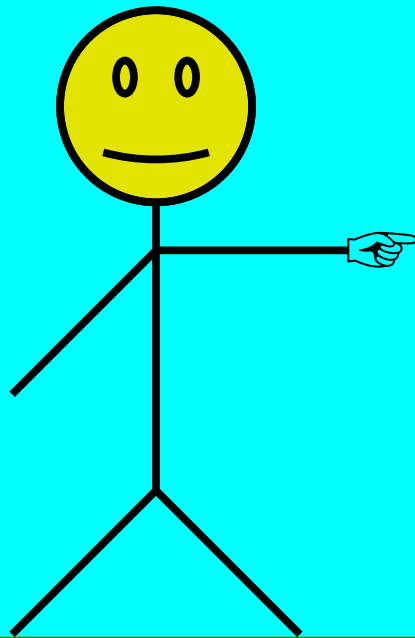


allocatedSize

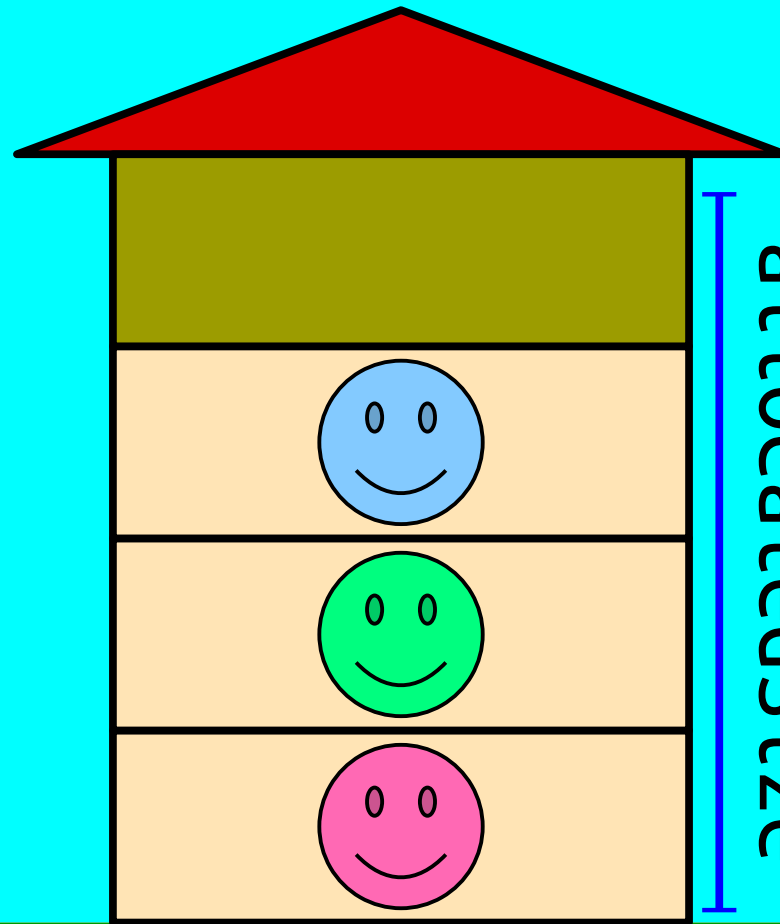
logicalSize



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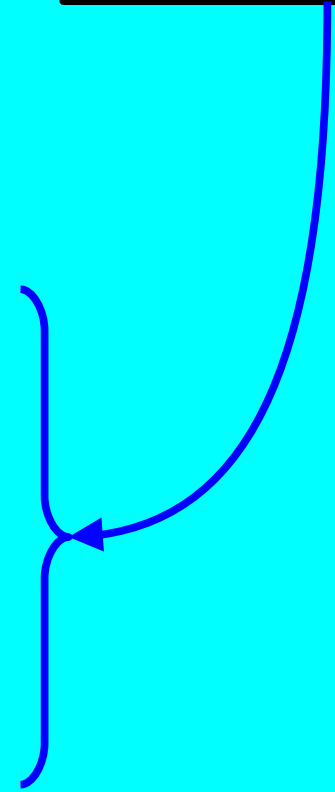


elems

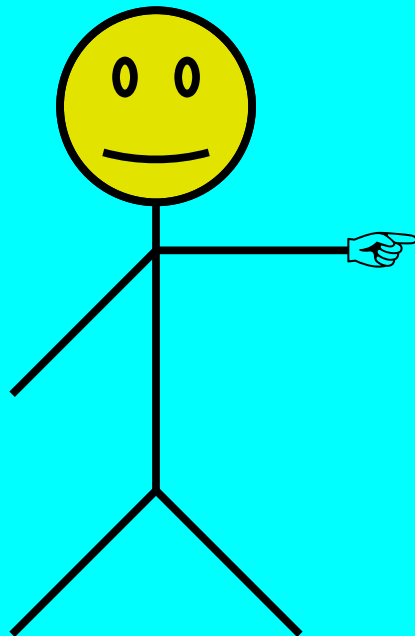


allocatedSize

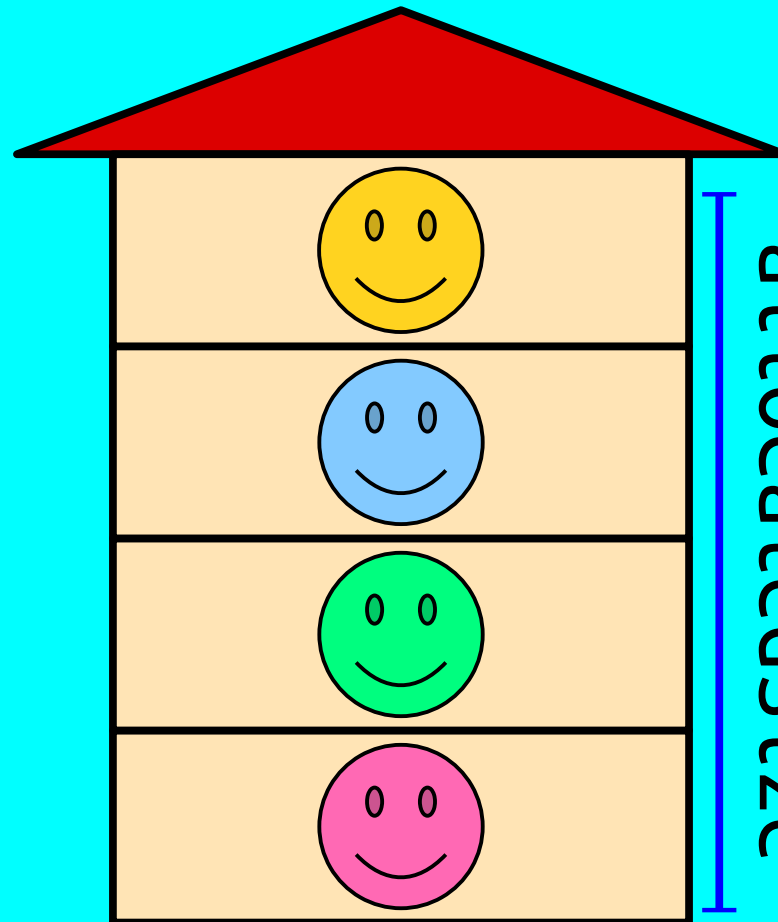
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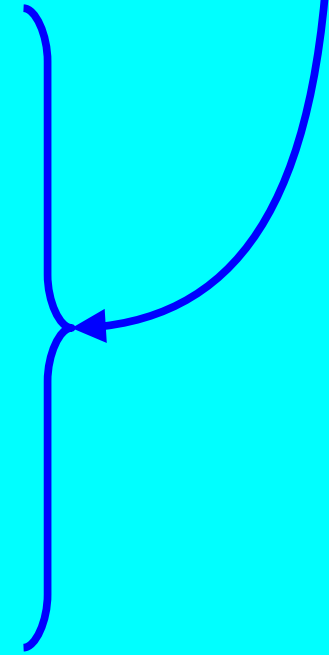


elems

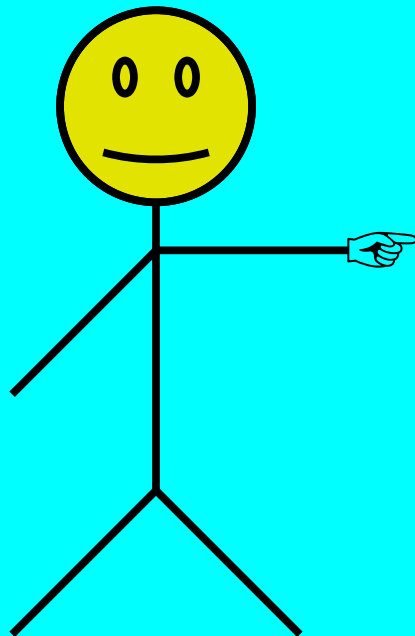


allocatedSize

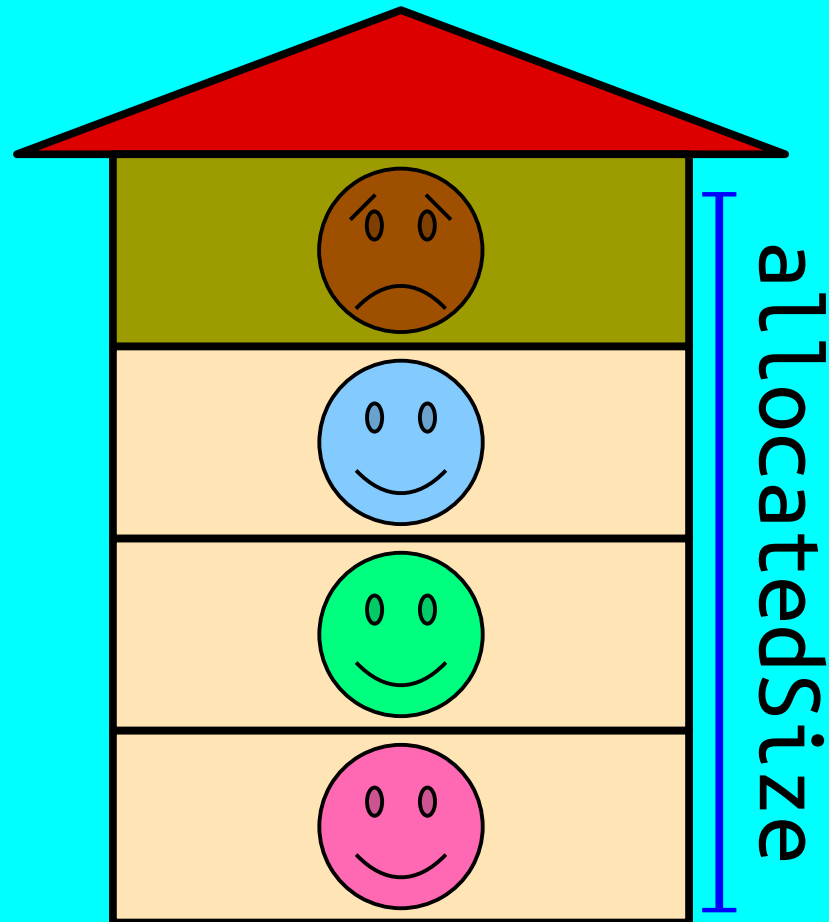
logicalSize



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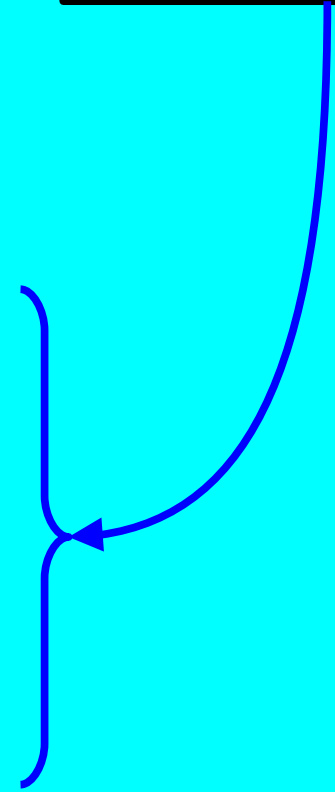


elems

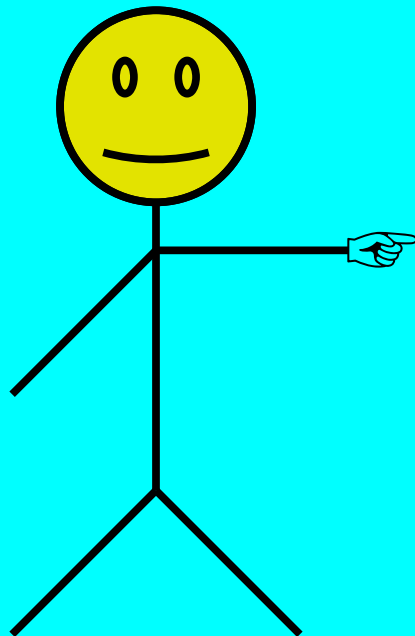


allocatedSize

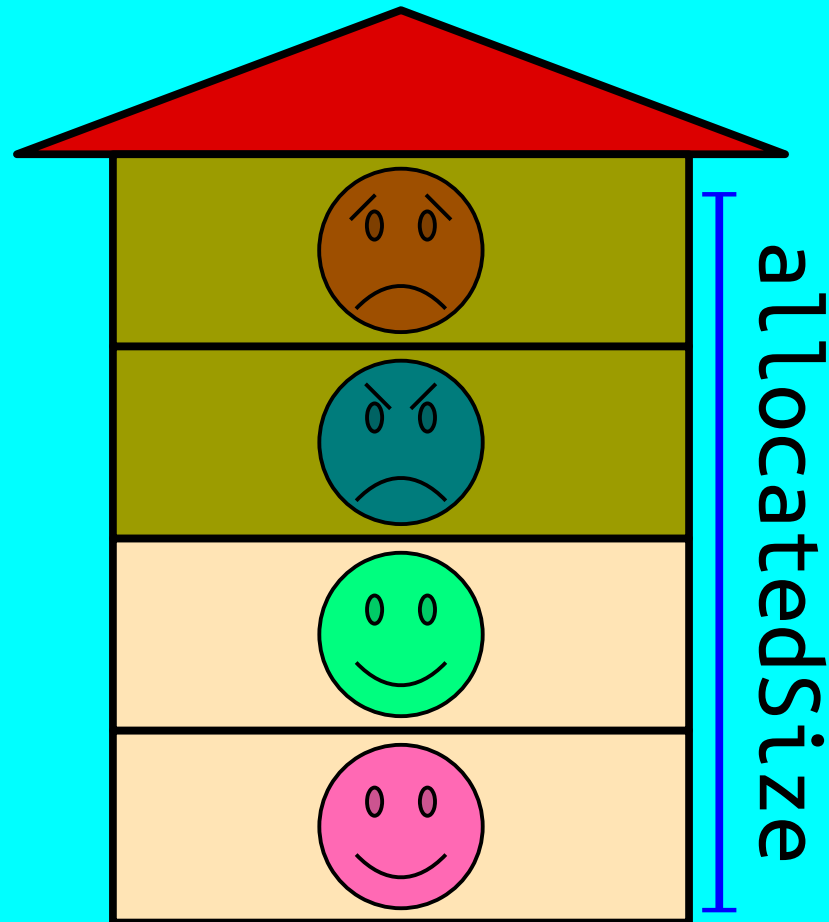
logicalSize



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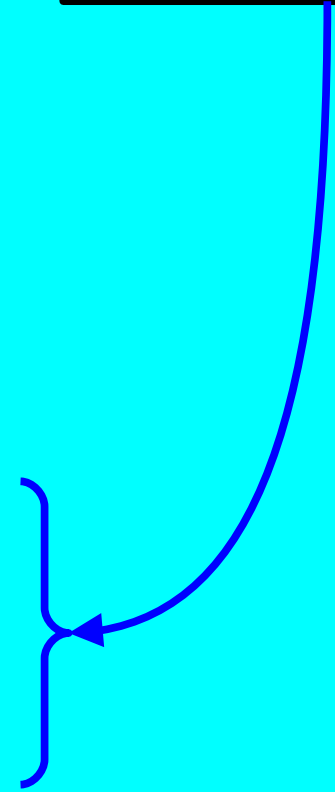


elems

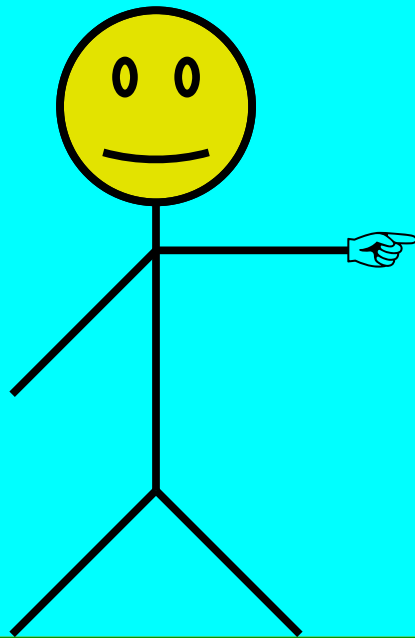


allocatedSize

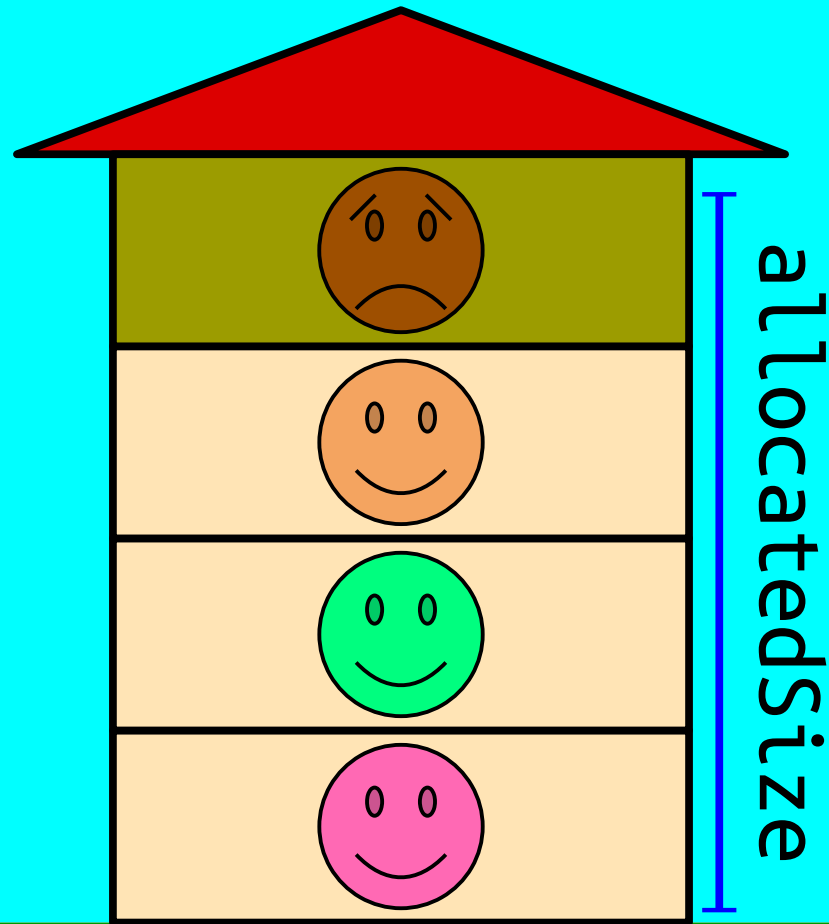
logicalSize



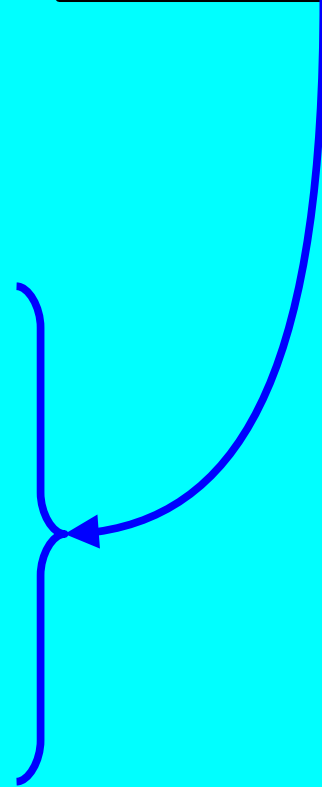
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```



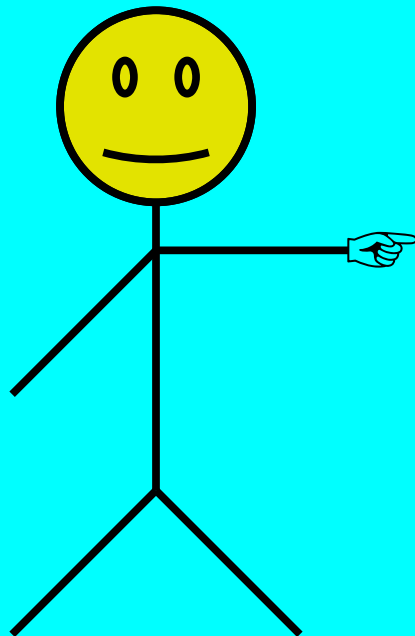
elems



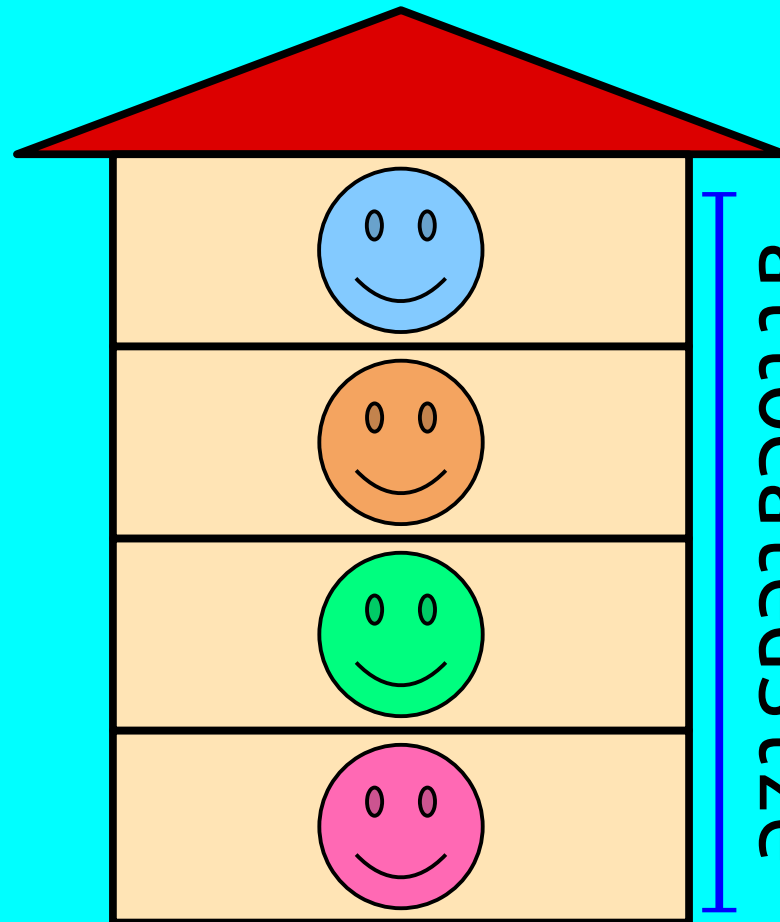
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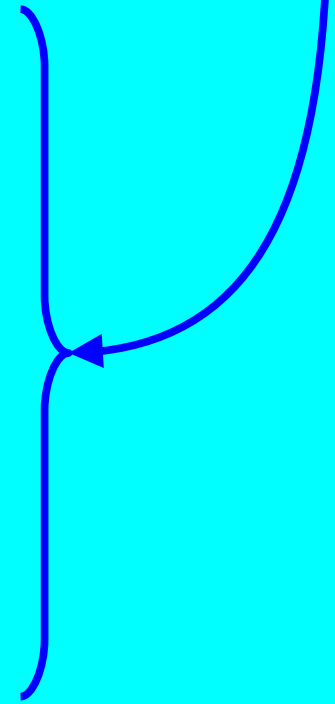


elems



allocatedSize

logicalSize



```
class OurStack {
public:
    OurStack();

    void push(int value);
    int peek() const;
    int pop();

    int size() const;
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private:
    int* elems;
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private:  
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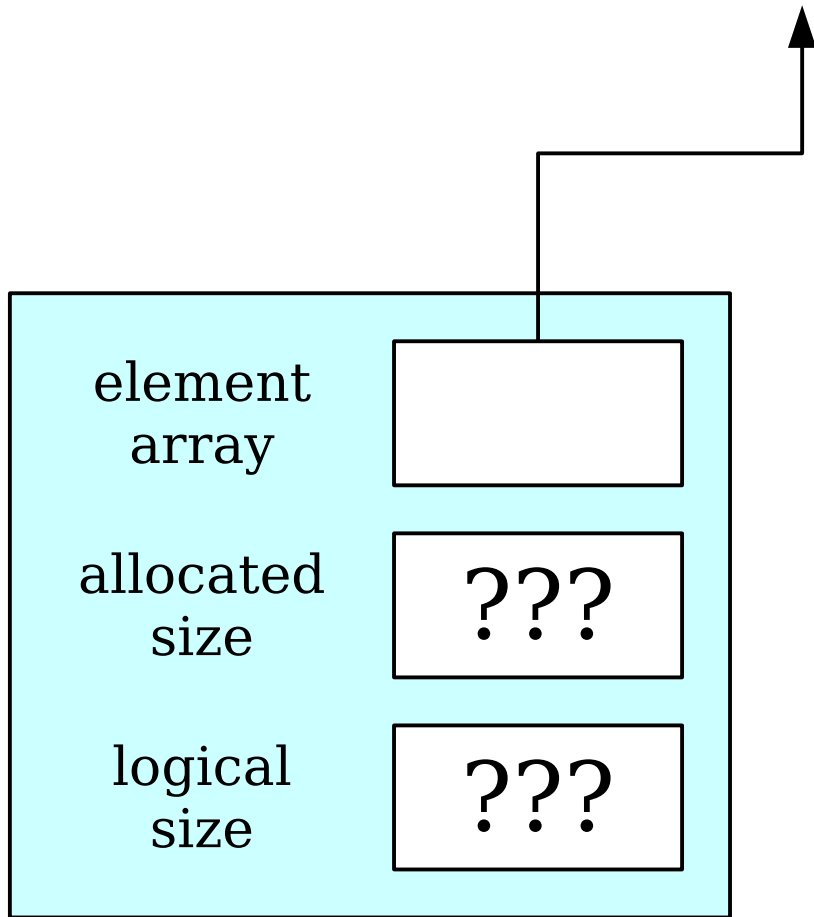
Cradle to Grave

```
int main() {  
    OurStack stack;  
  
    /* The stack lives a rich, happy,  
     * fulfilling life, the kind we  
     * all aspire to.  
     */  
  
    return 0;  
}
```


Cradle to Grave

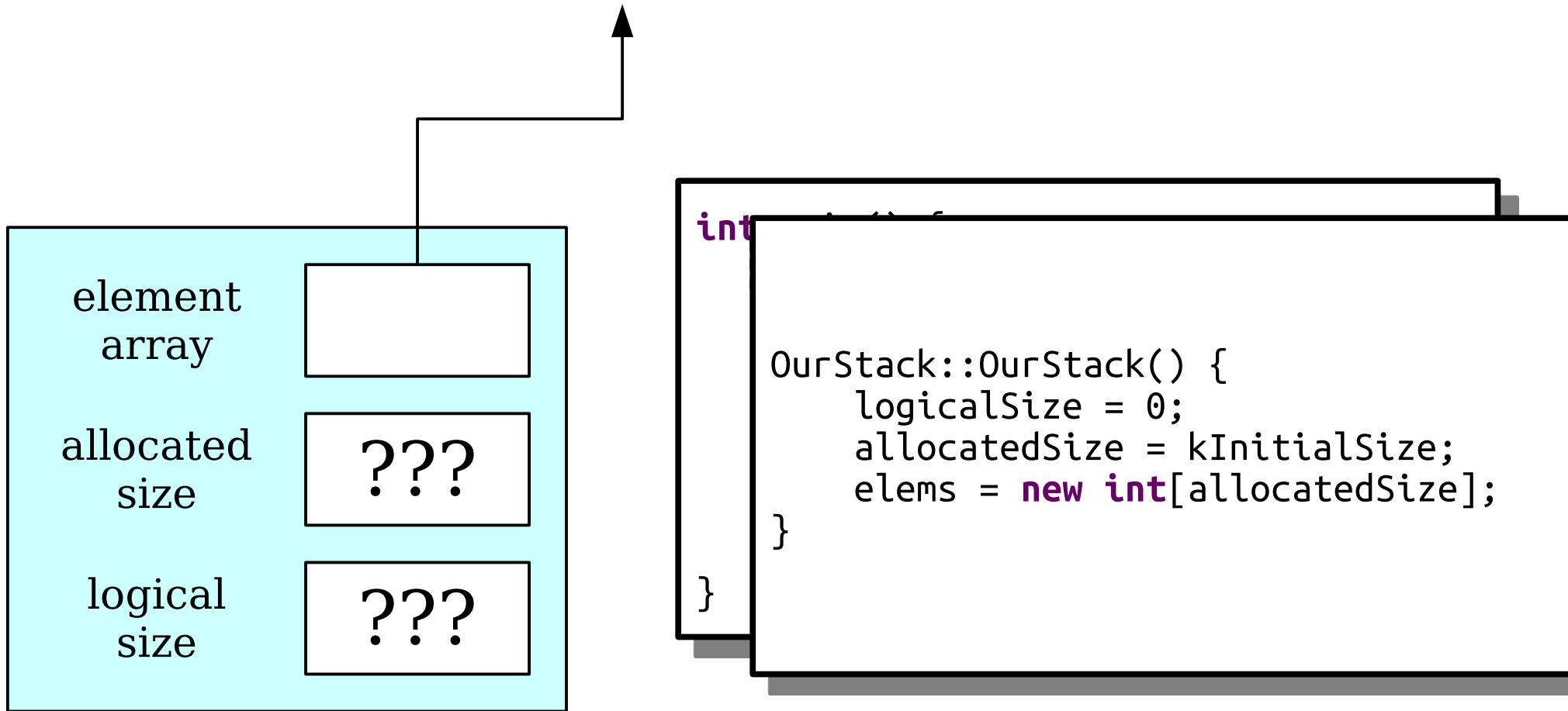
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Cradle to Grave

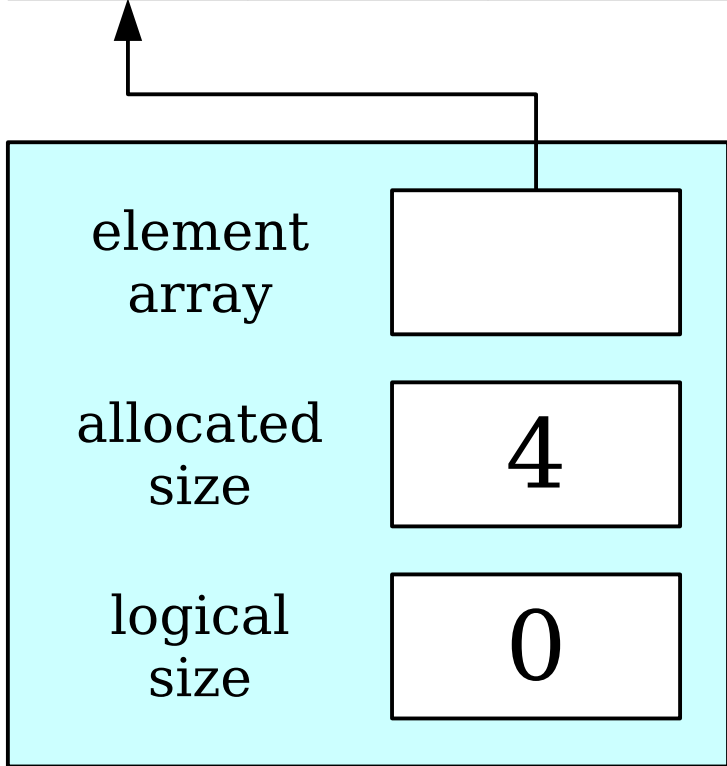
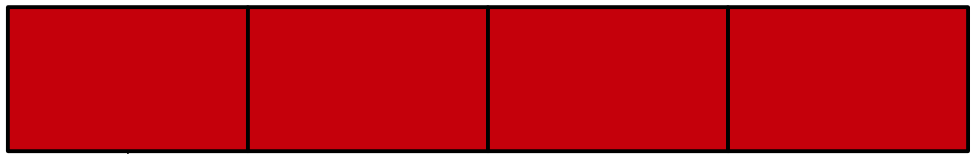


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Cradle to Grave

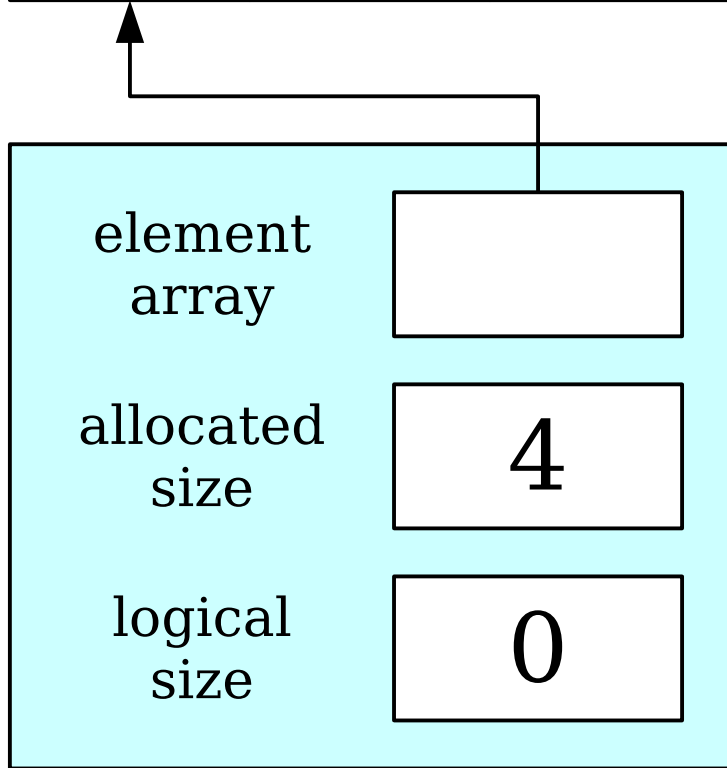
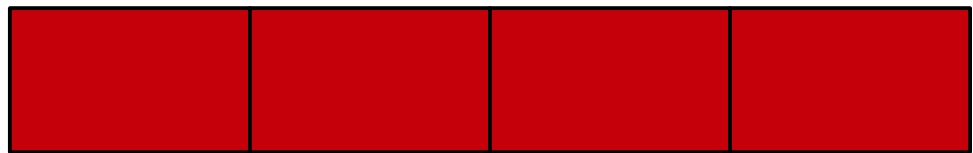


Cradle to Grave



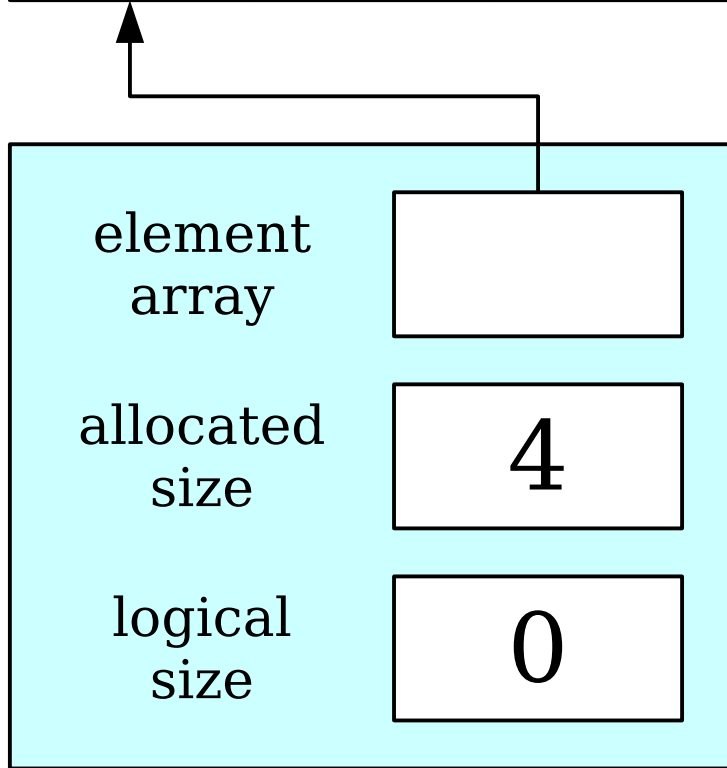
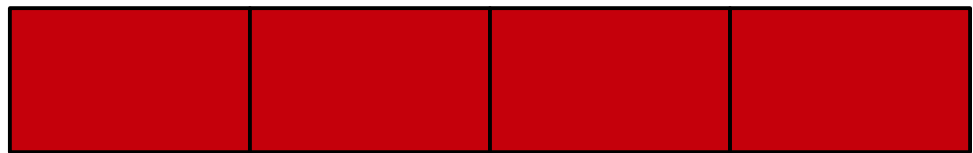
```
int OurStack() {  
    OurStack::OurStack() {  
        logicalSize = 0;  
        allocatedSize = kInitialSize;  
        elems = new int[allocatedSize];  
    }  
}
```

Cradle to Grave



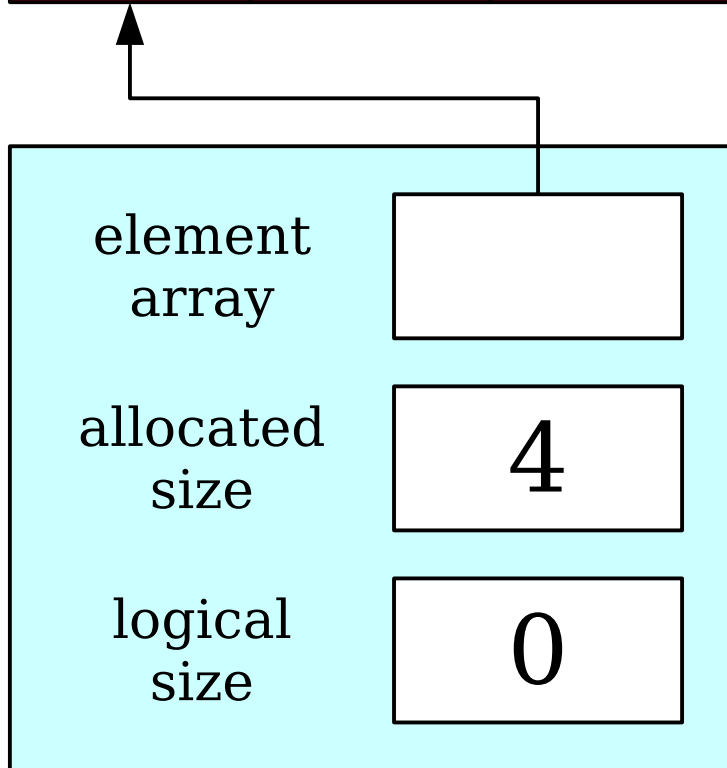
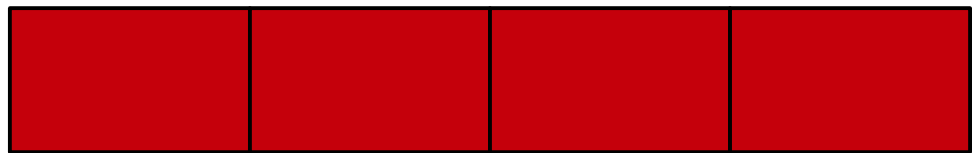
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Cradle to Grave



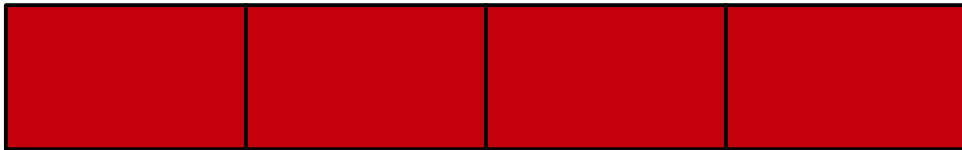
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int main() {  
    OurStack stack;  
  
    /* The stack lives a rich, happy,  
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     */  
  
    return 0;  
}
```

Cradle to Grave



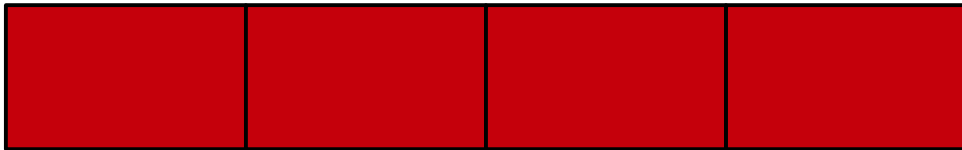
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Cradle to Grave



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    return 0;  
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```


Cradle to Grave



**Memory
Leak!**

```
int main() {  
    OurStack stack;  
  
    /* The stack lives a rich, happy,  
     * fulfilling life, the kind we  
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     */  
  
    return 0;  
}
```

Cleaning Up our Messes

Destructors

- A **destructor** is a special member function responsible for cleaning up an object's memory.
- It's automatically called whenever an object's lifetime ends (for example, if it's a local variable that goes out of scope.)
- The destructor for a class named **ClassName** has signature

~ClassName();

```
class OurStack {
public:
    OurStack();
    ~OurStack();

    void push(int value);
    int peek() const;
    int pop();

    int size() const;
    bool isEmpty() const;

private:
    int* elems;
    int allocatedSize;
    int logicalSize;
};
```

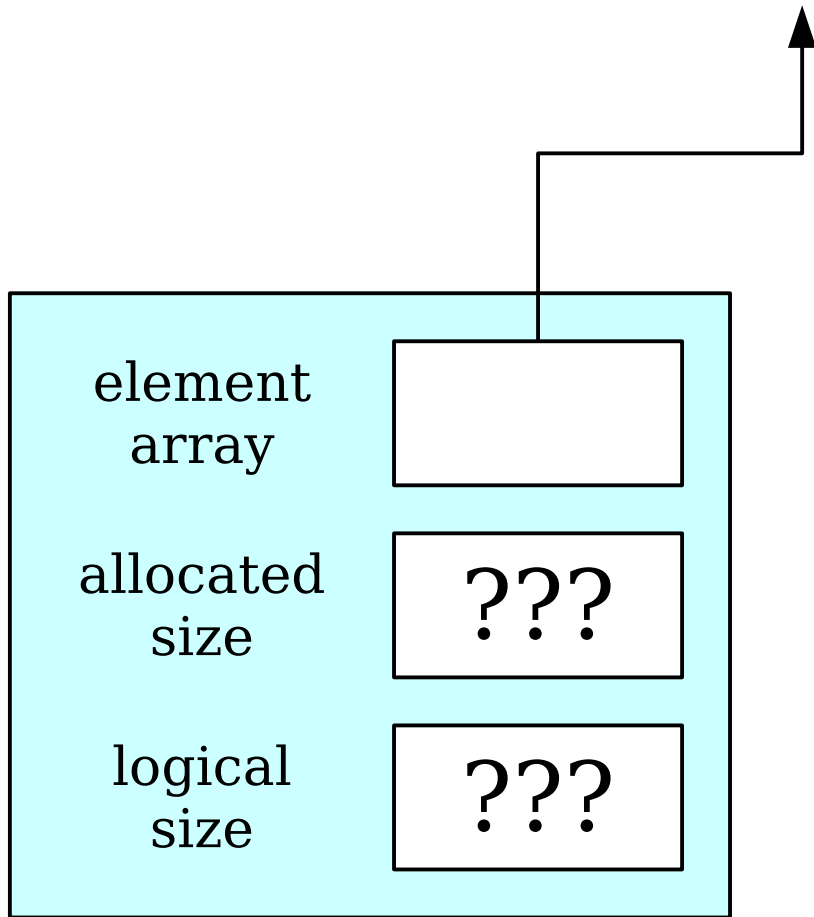
Cradle to Grave

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    return 0;  
}
```

Cradle to Grave

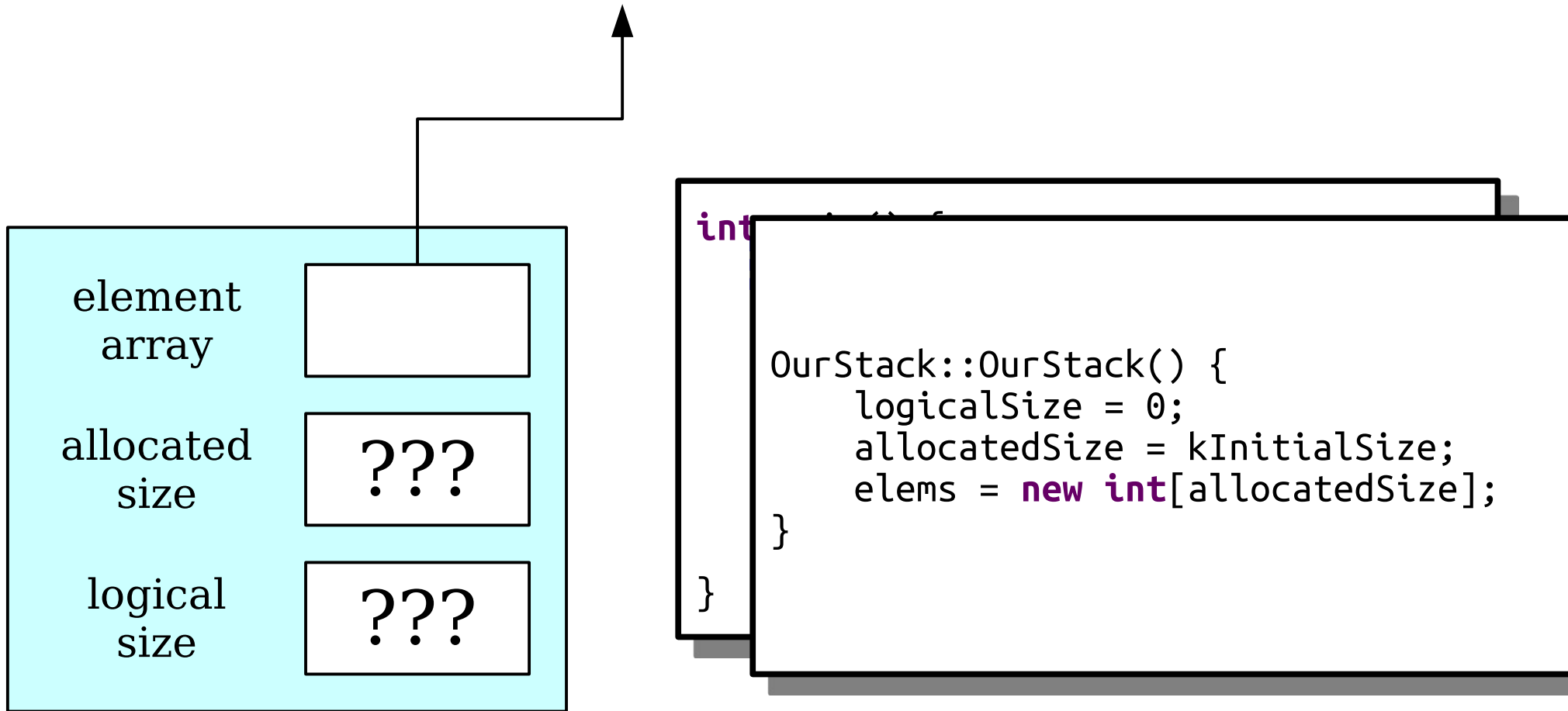
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    OurStack stack;  
  
    /* The stack lives a rich, happy,  
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     */  
  
    return 0;  
}
```

Cradle to Grave

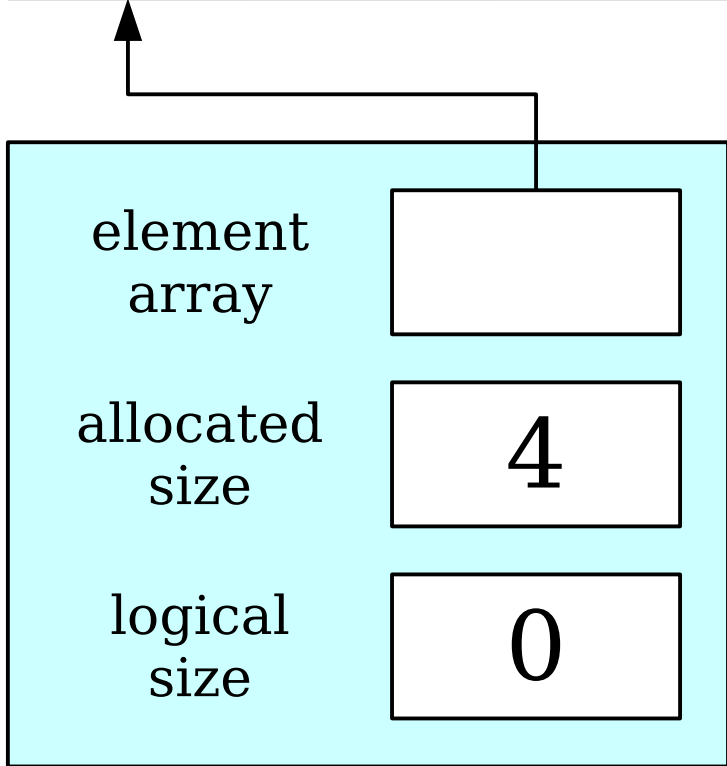
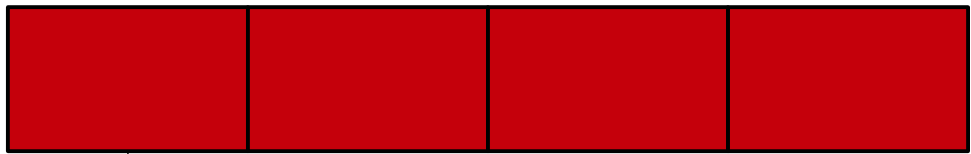


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    OurStack stack;  
  
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     */  
  
    return 0;  
}
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Cradle to Grave

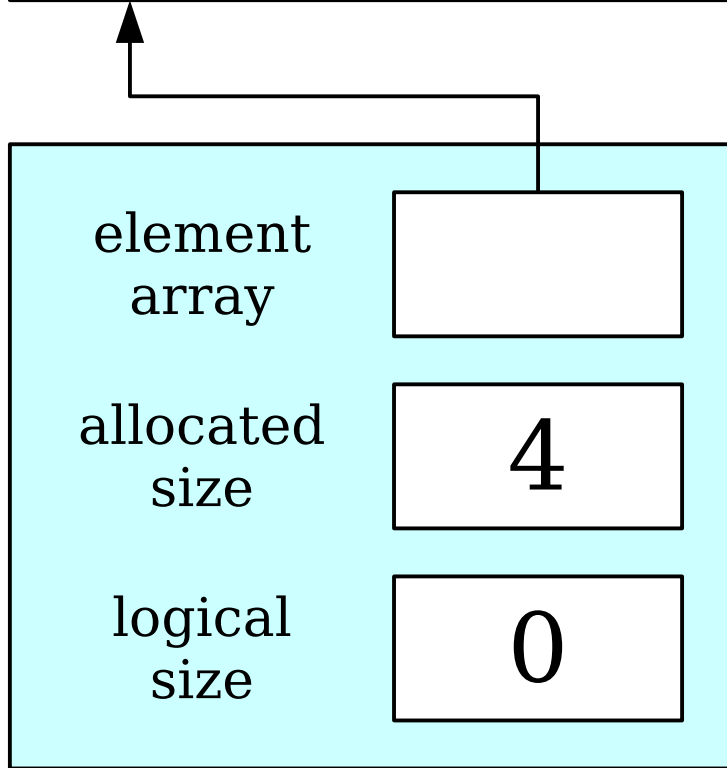
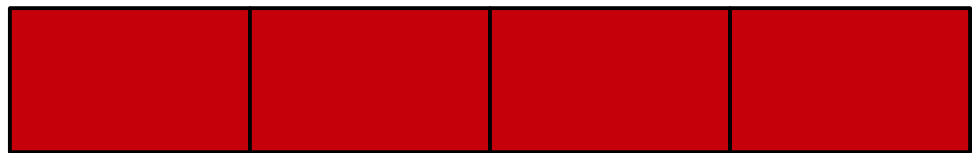


Cradle to Grave



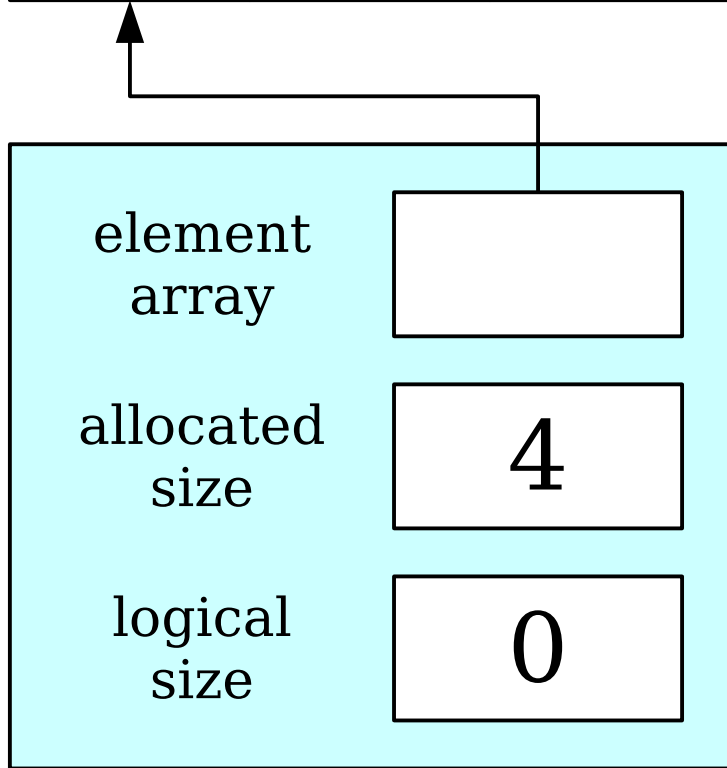
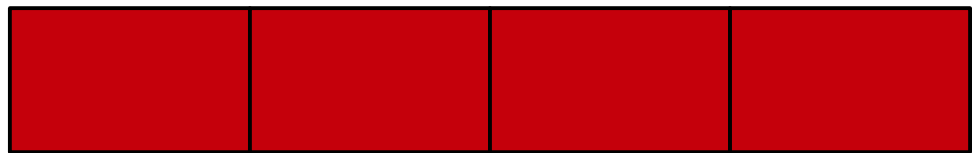
```
int OurStack() {  
    OurStack::OurStack() {  
        logicalSize = 0;  
        allocatedSize = kInitialSize;  
        elems = new int[allocatedSize];  
    }  
}
```


Cradle to Grave



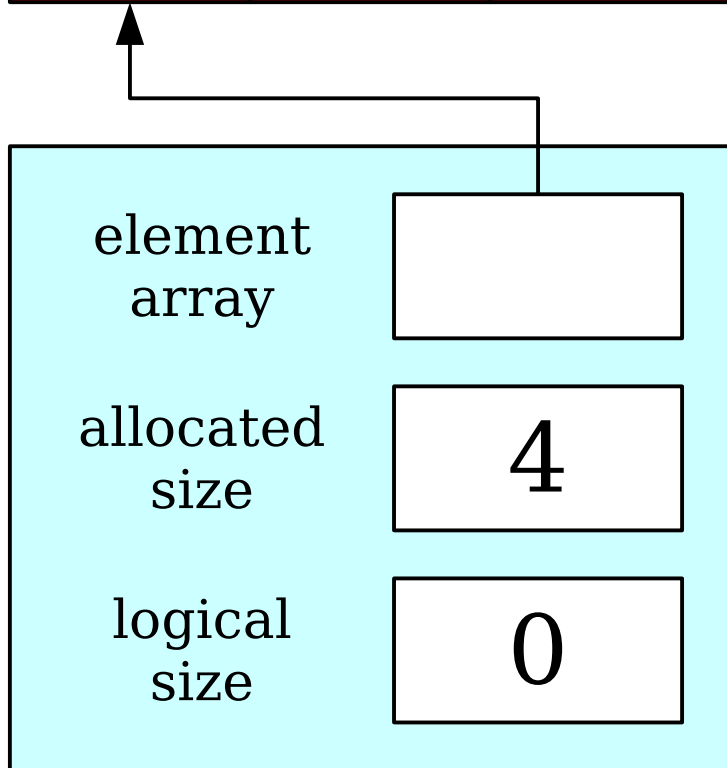
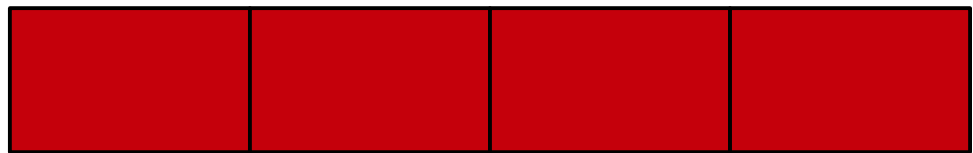
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     */  
  
    return 0;  
}
```

Cradle to Grave



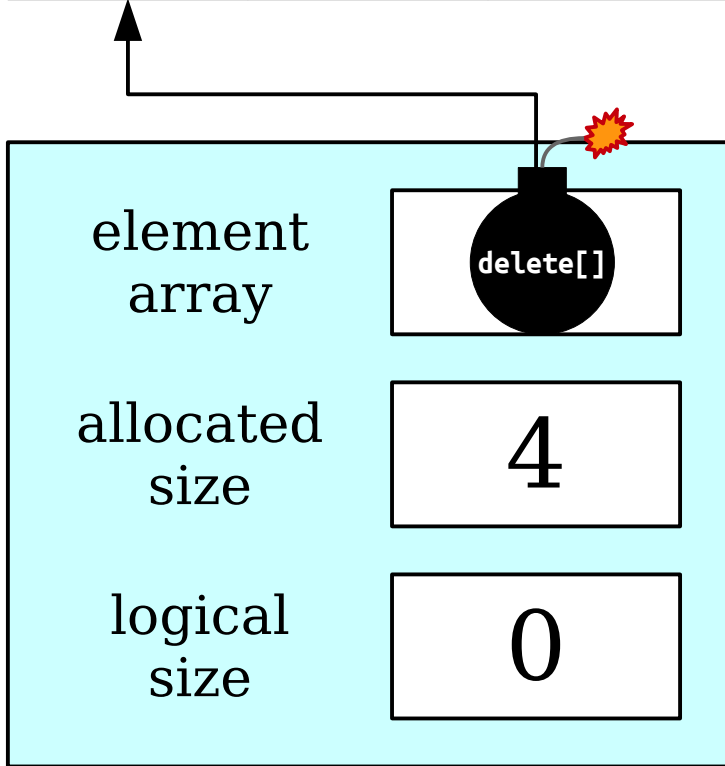
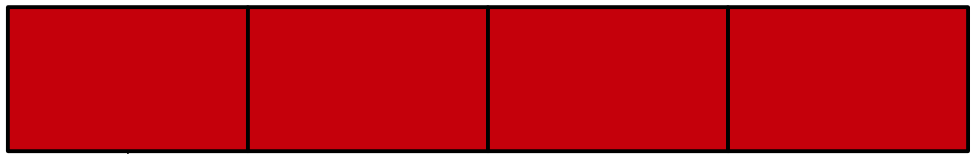
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    return 0;  
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Cradle to Grave



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int main() {  
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     */  
    return 0;  
}
```


Cradle to Grave



```
int main() {  
  
    OurStack::~~OurStack() {  
        delete[] elems;  
    }  
}
```

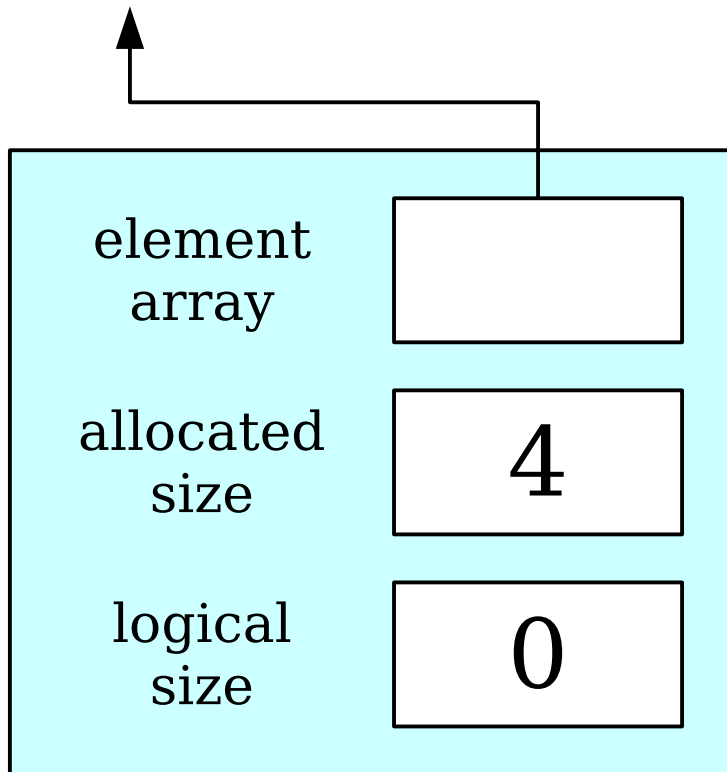
Cradle to Grave

**Dynamic
Deallocation!**

element array	
allocated size	4
logical size	0

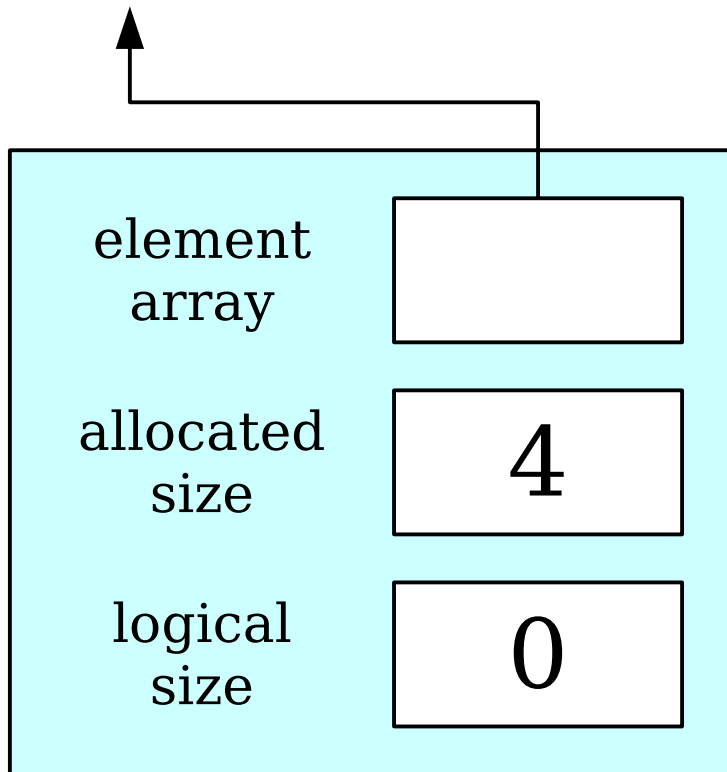
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    OurStack::~~OurStack() {  
        delete[] elems;  
    }  
}
```

Cradle to Grave



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Cradle to Grave

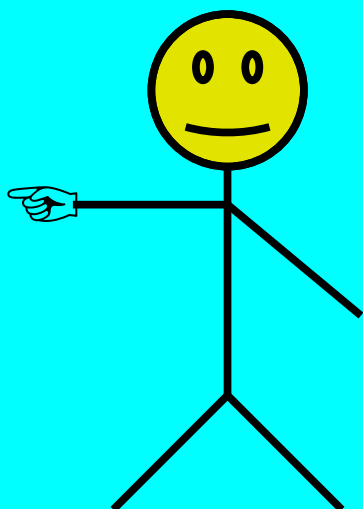
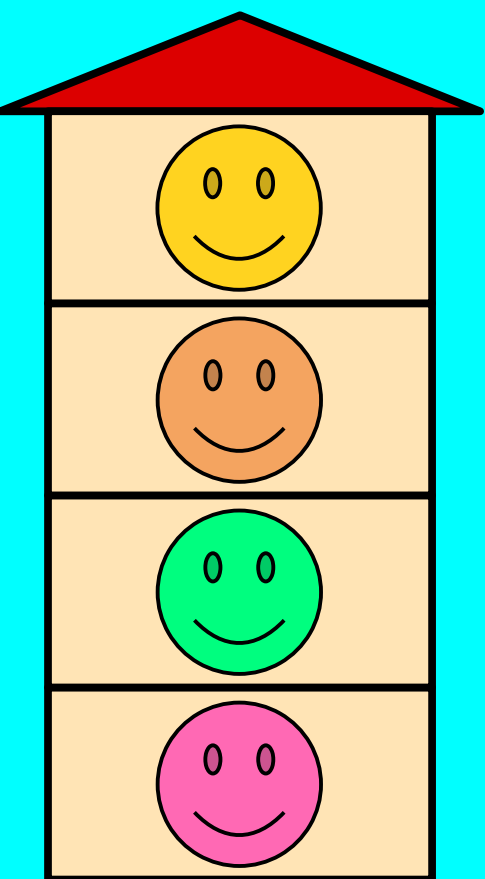


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Cradle to Grave

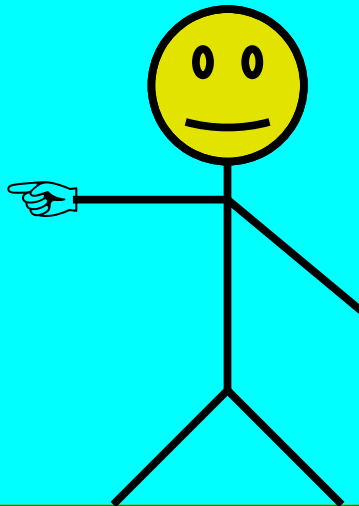
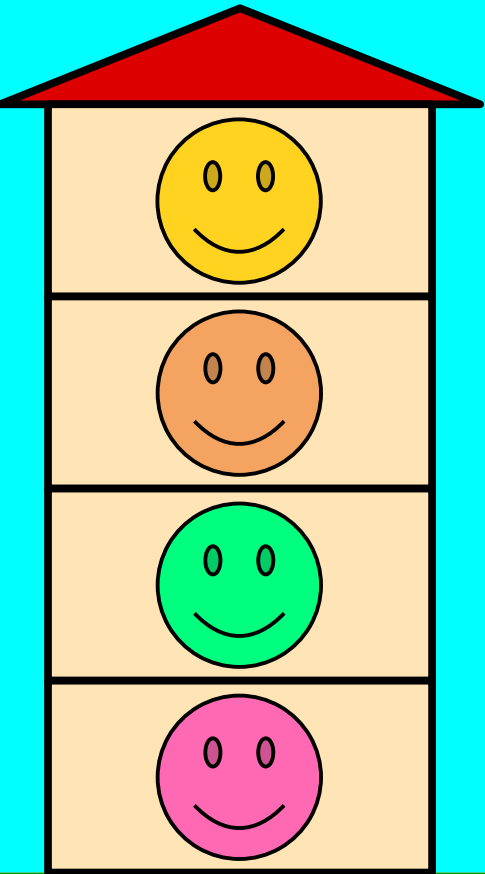
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    OurStack stack;  
  
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     */  
    return 0;  
}
```


Getting More Space

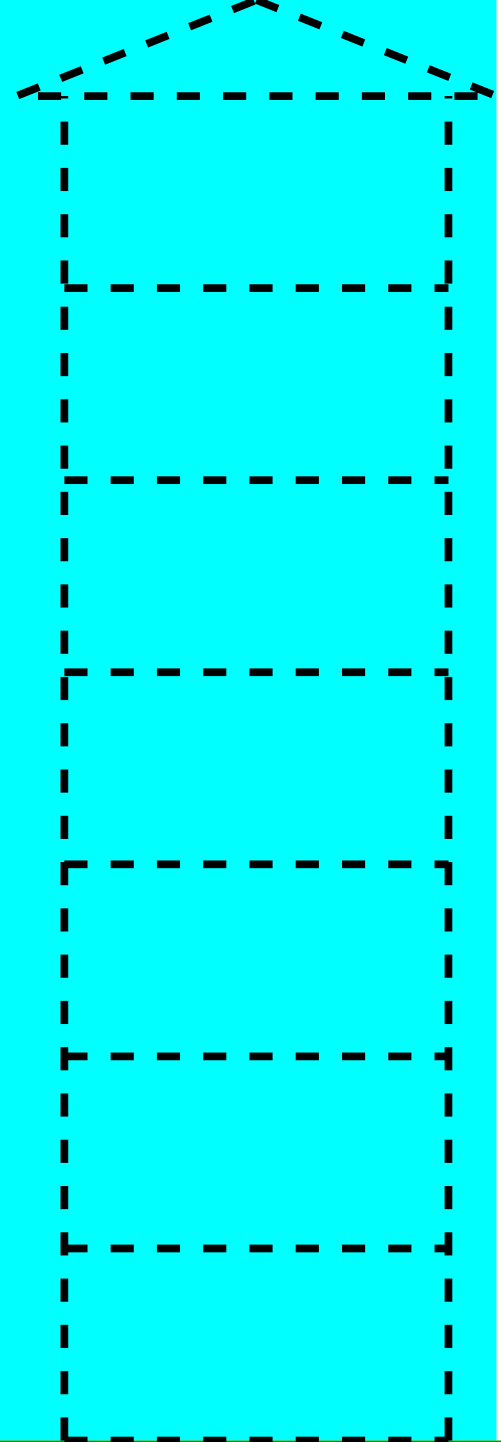


eLems

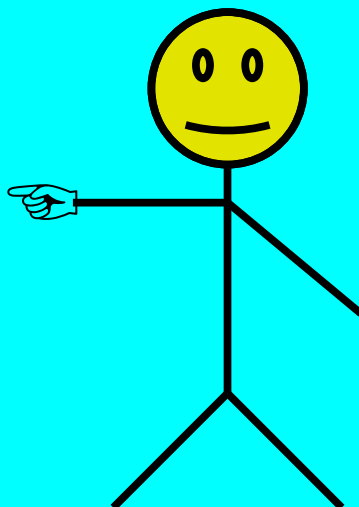
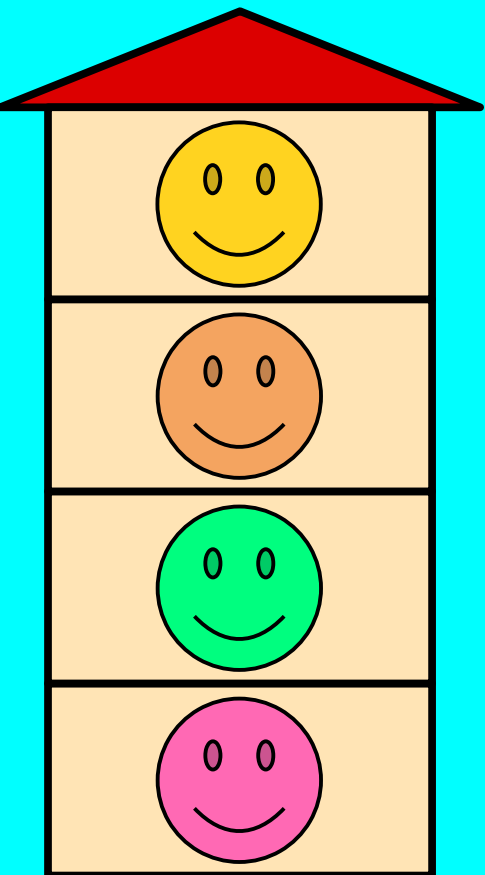
```
allocatedSize = /* bigger */;
```



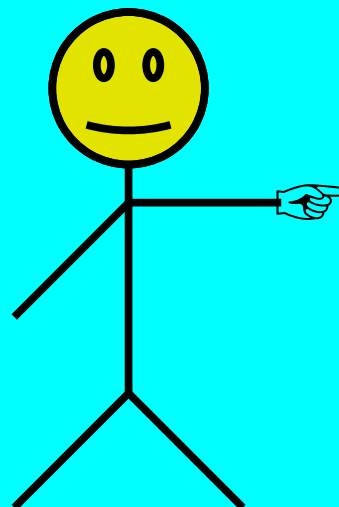
e1ems



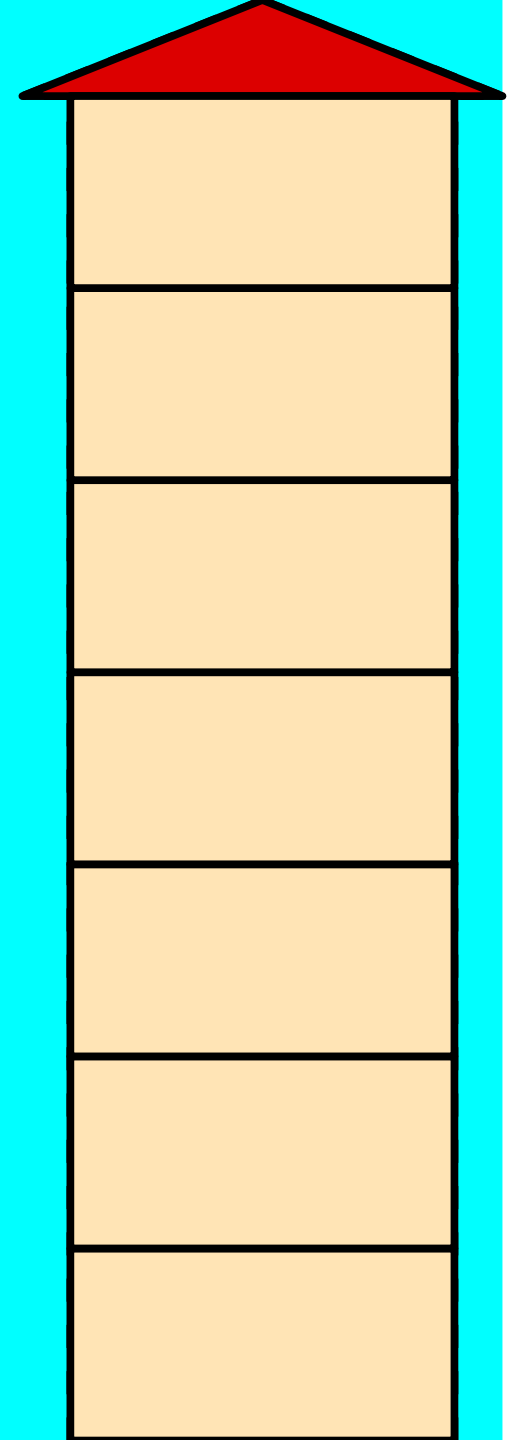
```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];
```



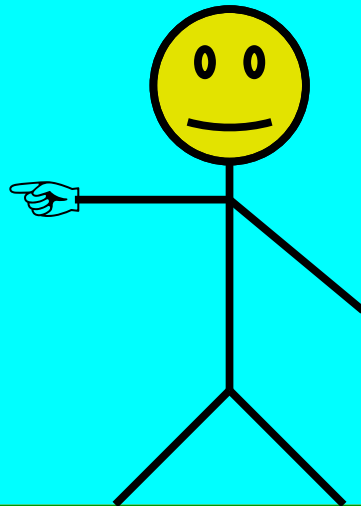
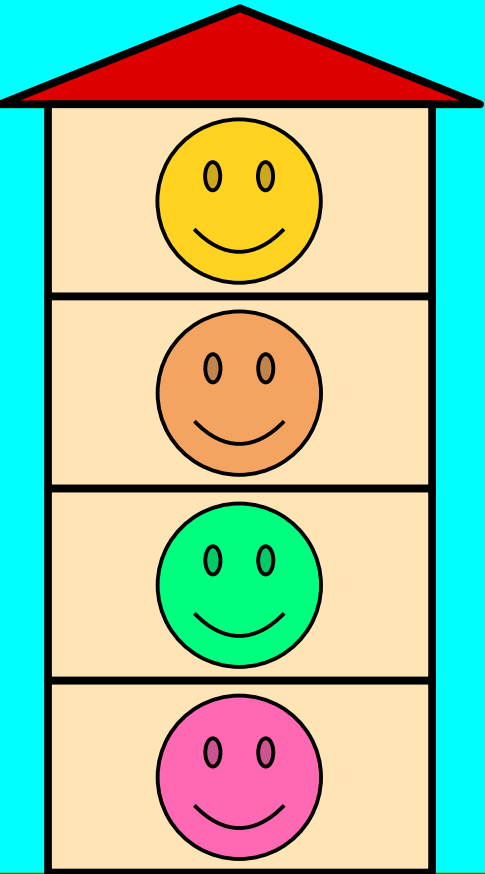
elems



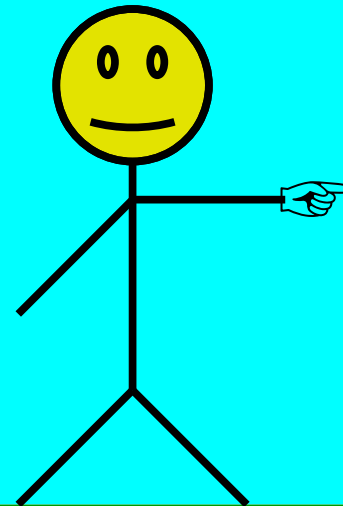
helper



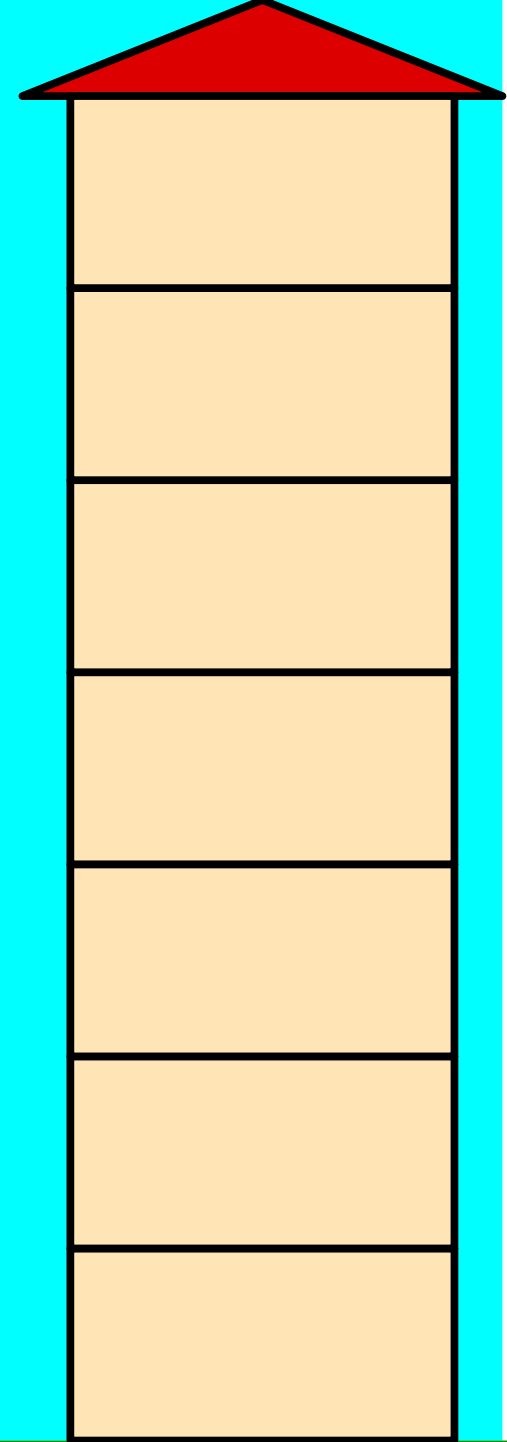
```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];  
/* ... move elements over ... */
```



elems

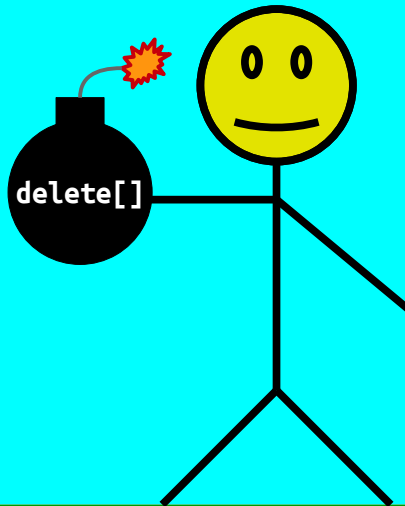


helper

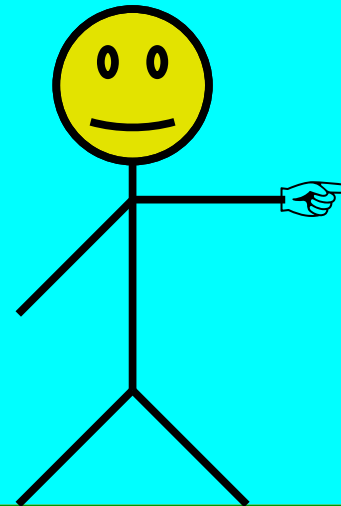


```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];  
/* ... move elements over ... */  
delete[] elems;
```

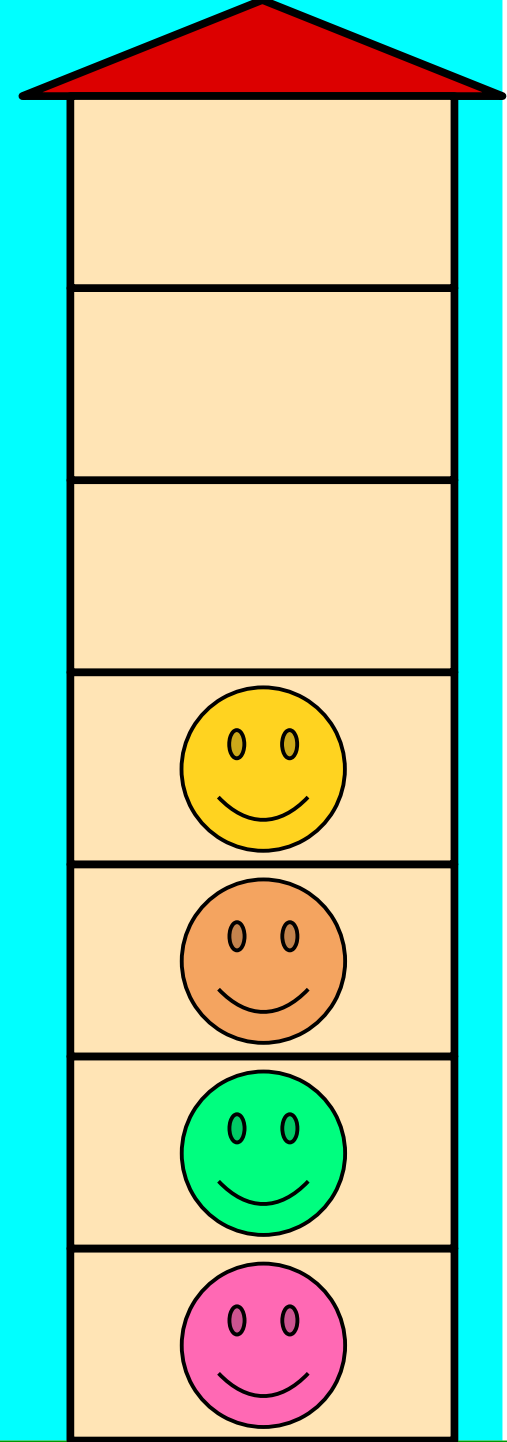
*Dynamic
Deallocation!*



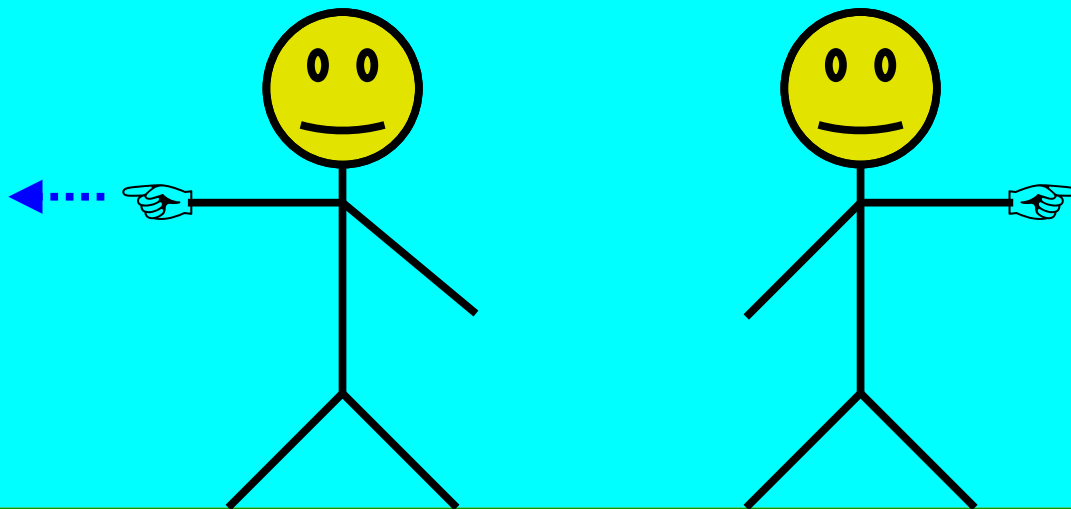
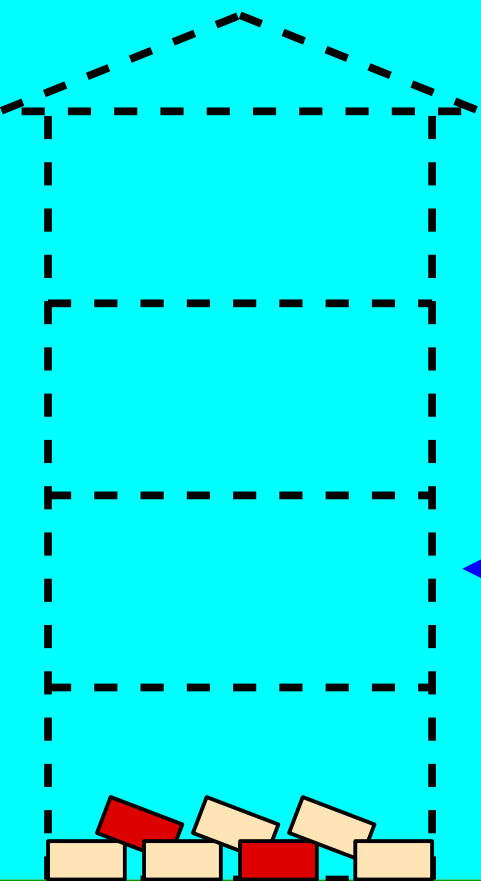
elems



helper

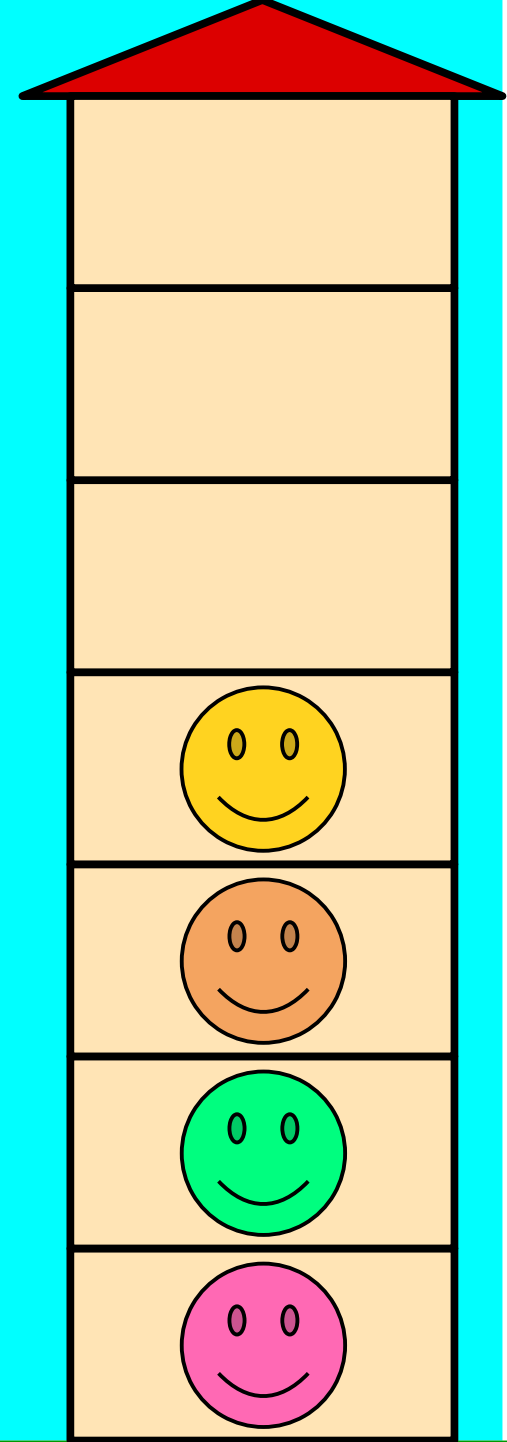


```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];  
/* ... move elements over ... */  
delete[] elems;
```

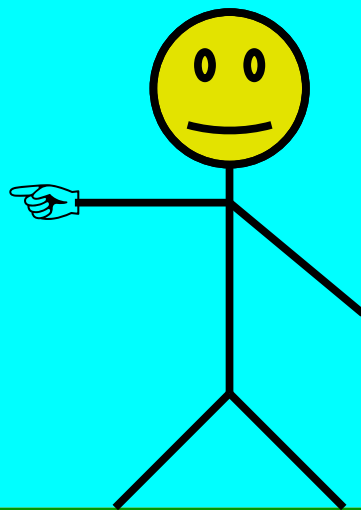
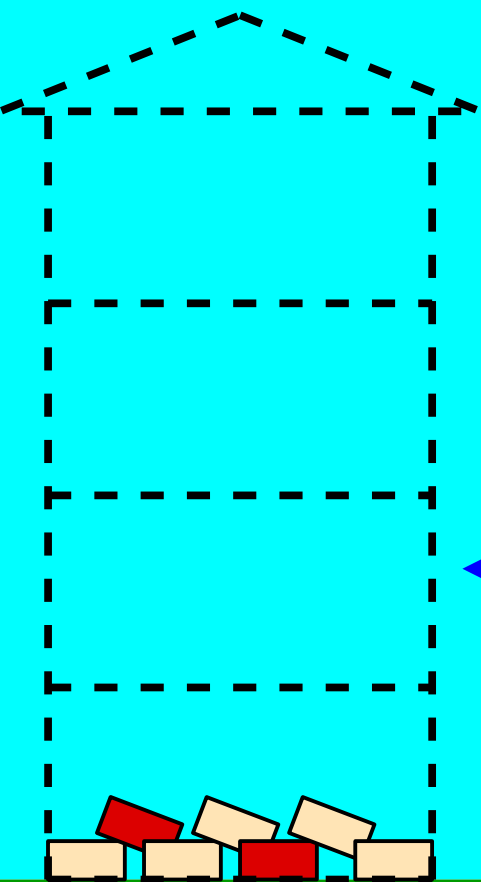


elems

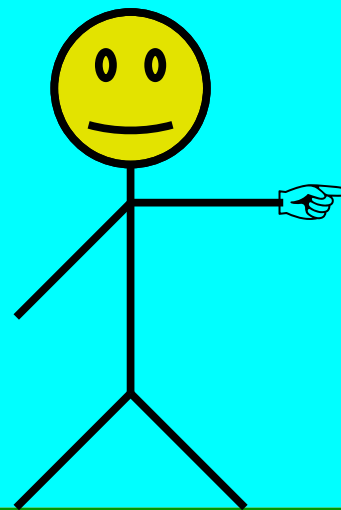
helper



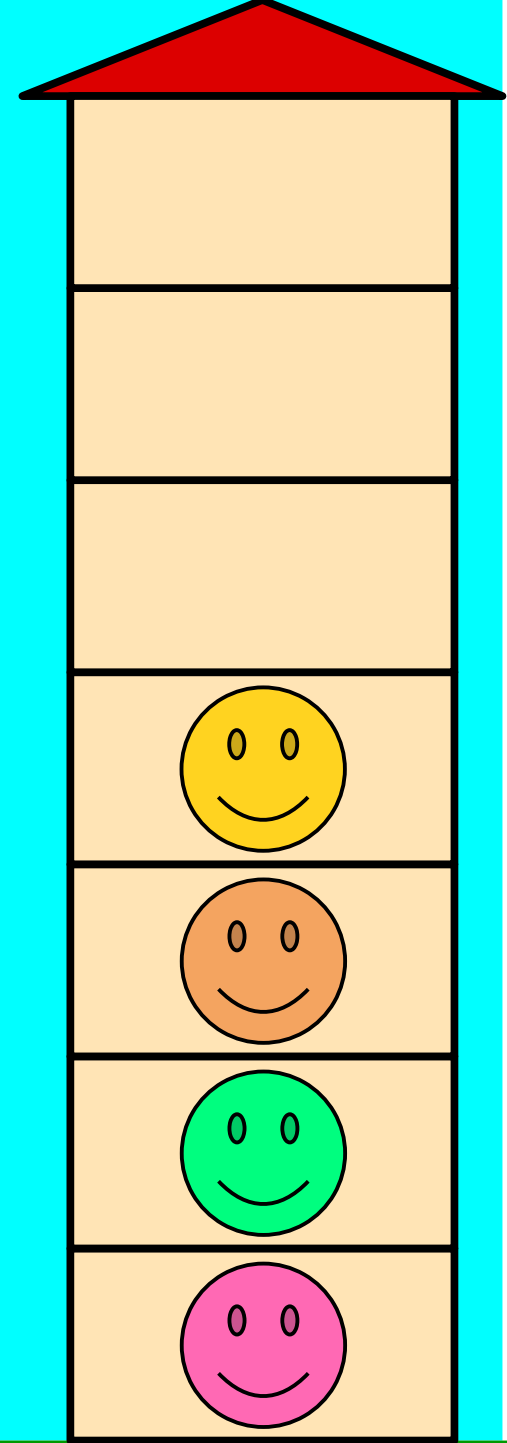
```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];  
/* ... move elements over ... */  
delete[] elems;  
elems = helper;
```



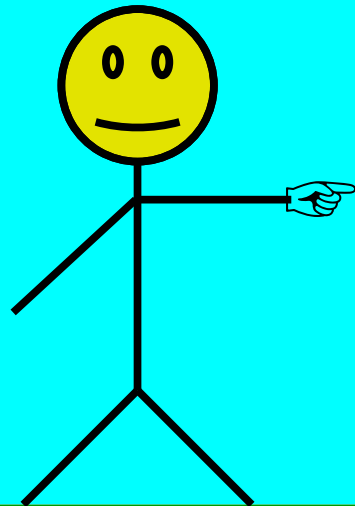
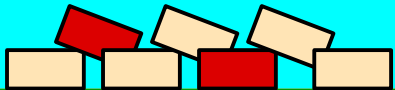
elems



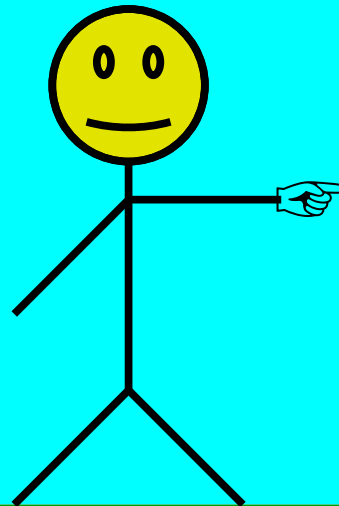
helper



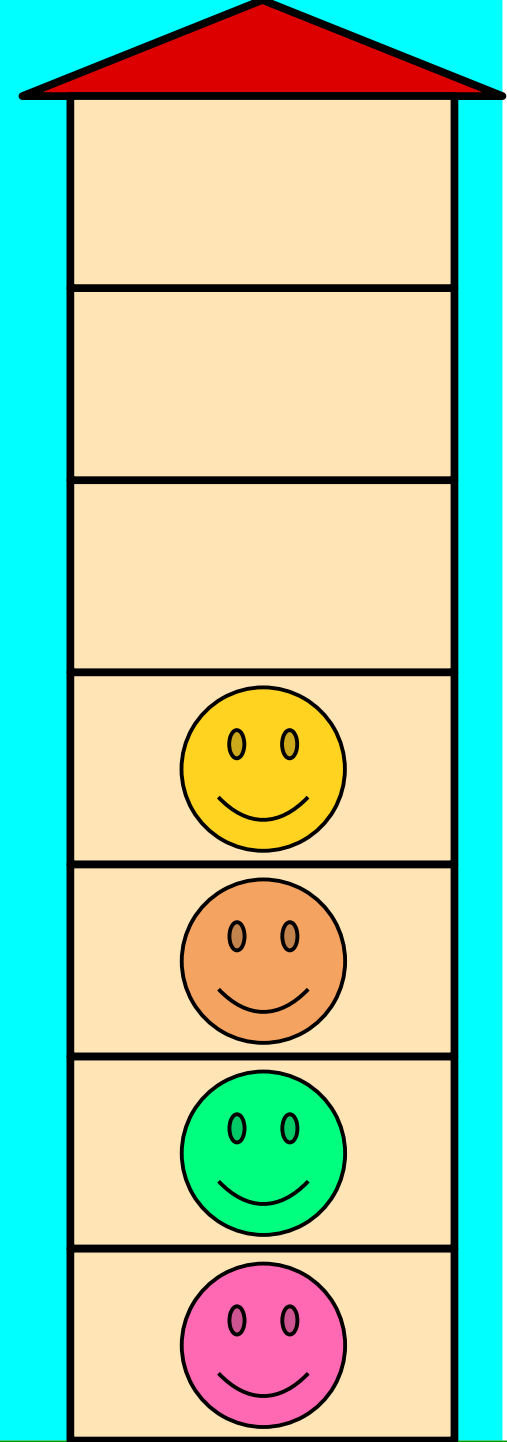

```
allocatedSize = /* bigger */;  
int* helper = new int[allocatedSize];  
/* ... move elements over ... */  
delete[] elems;  
elems = helper;
```



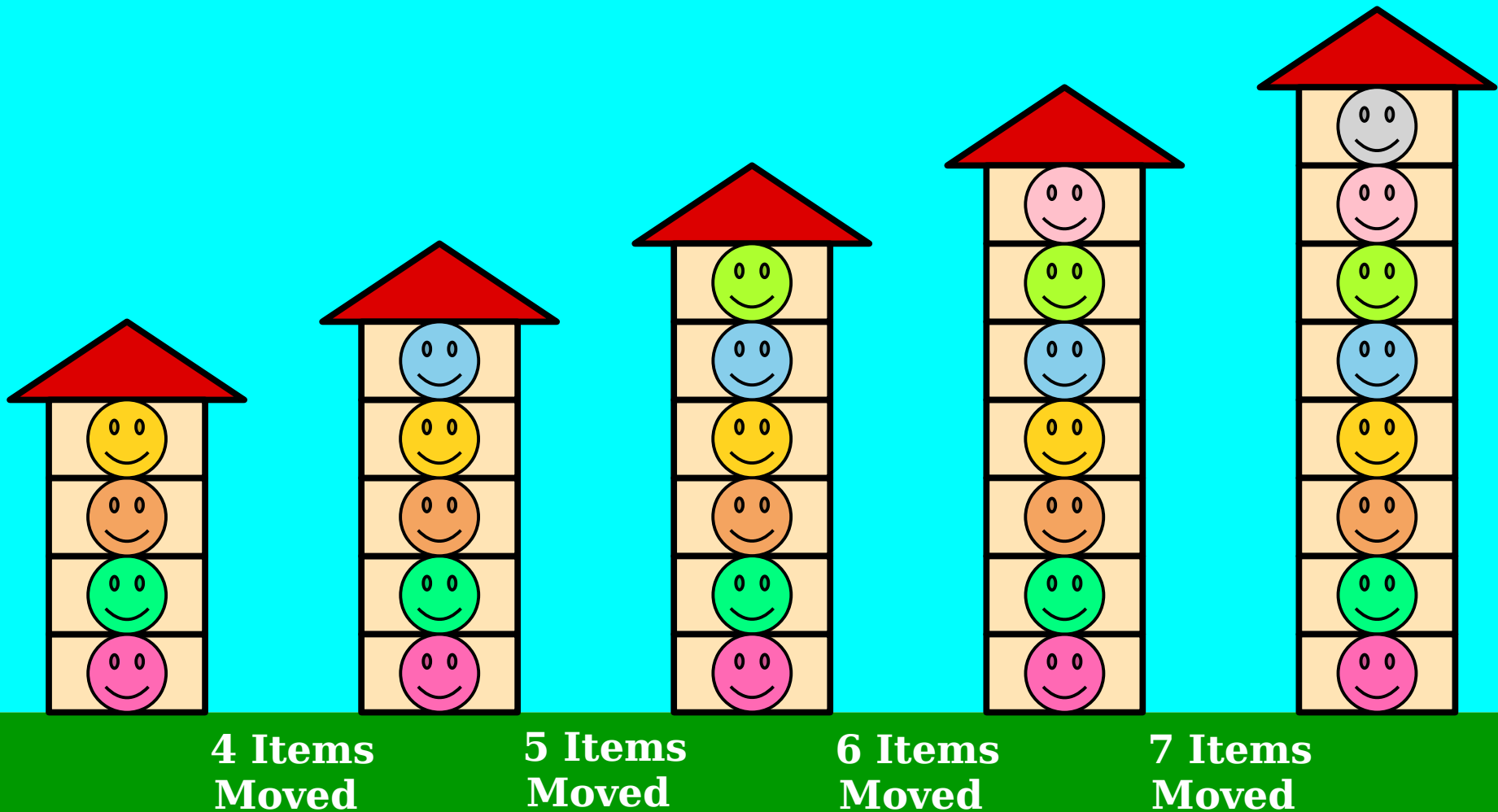
elems



helper

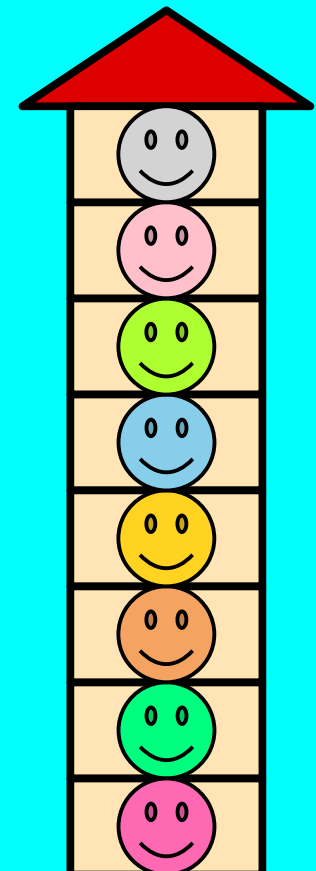


What is the big-O cost of a push?
What is the big-O cost of n pushes?



Every push beyond the first few requires moving all n elements from the old array to the new array.

Cost of a single push: $O(n)$.



4 Items
Moved

5 Items
Moved

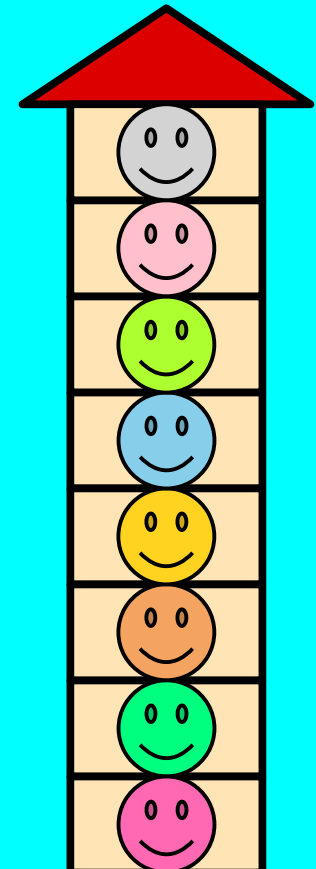
6 Items
Moved

7 Items
Moved

Every push beyond the first few requires moving all n elements from the old array to the new array.

Cost of doing n pushes:
 $4 + 5 + 6 + \dots + n = \mathbf{O(n^2)}$.

Question: How do we speed this up?

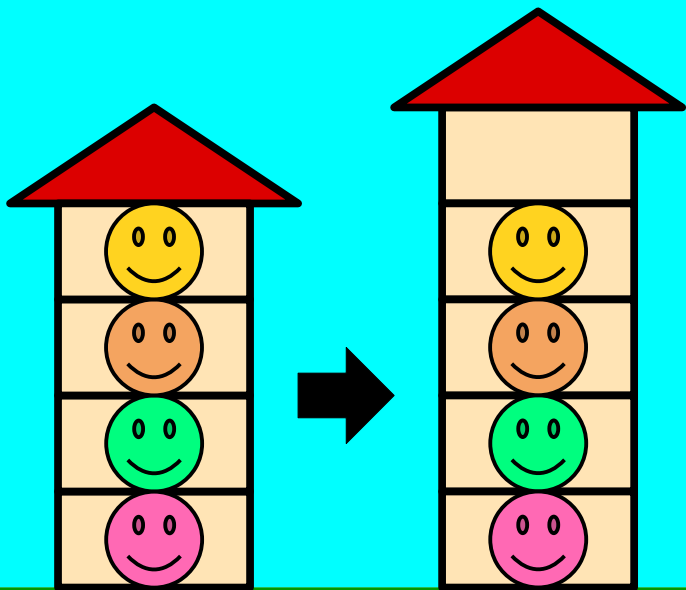


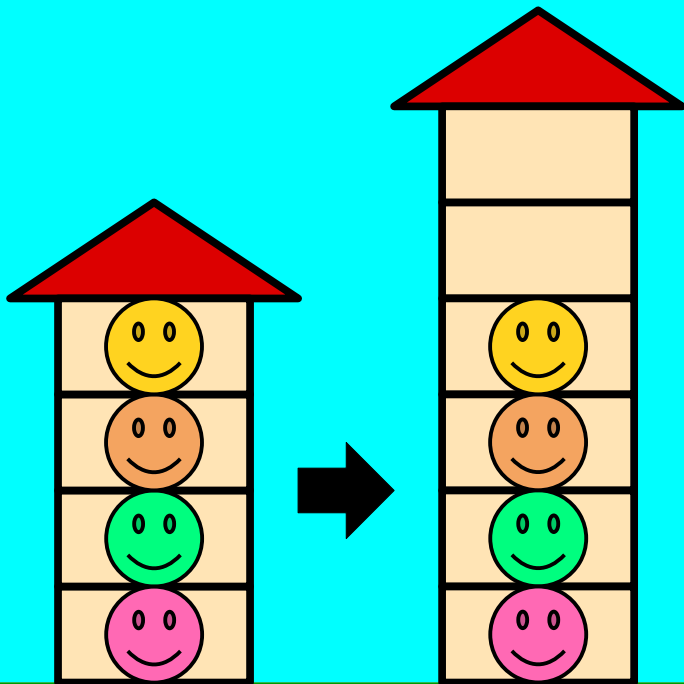
4 Items
Moved

5 Items
Moved

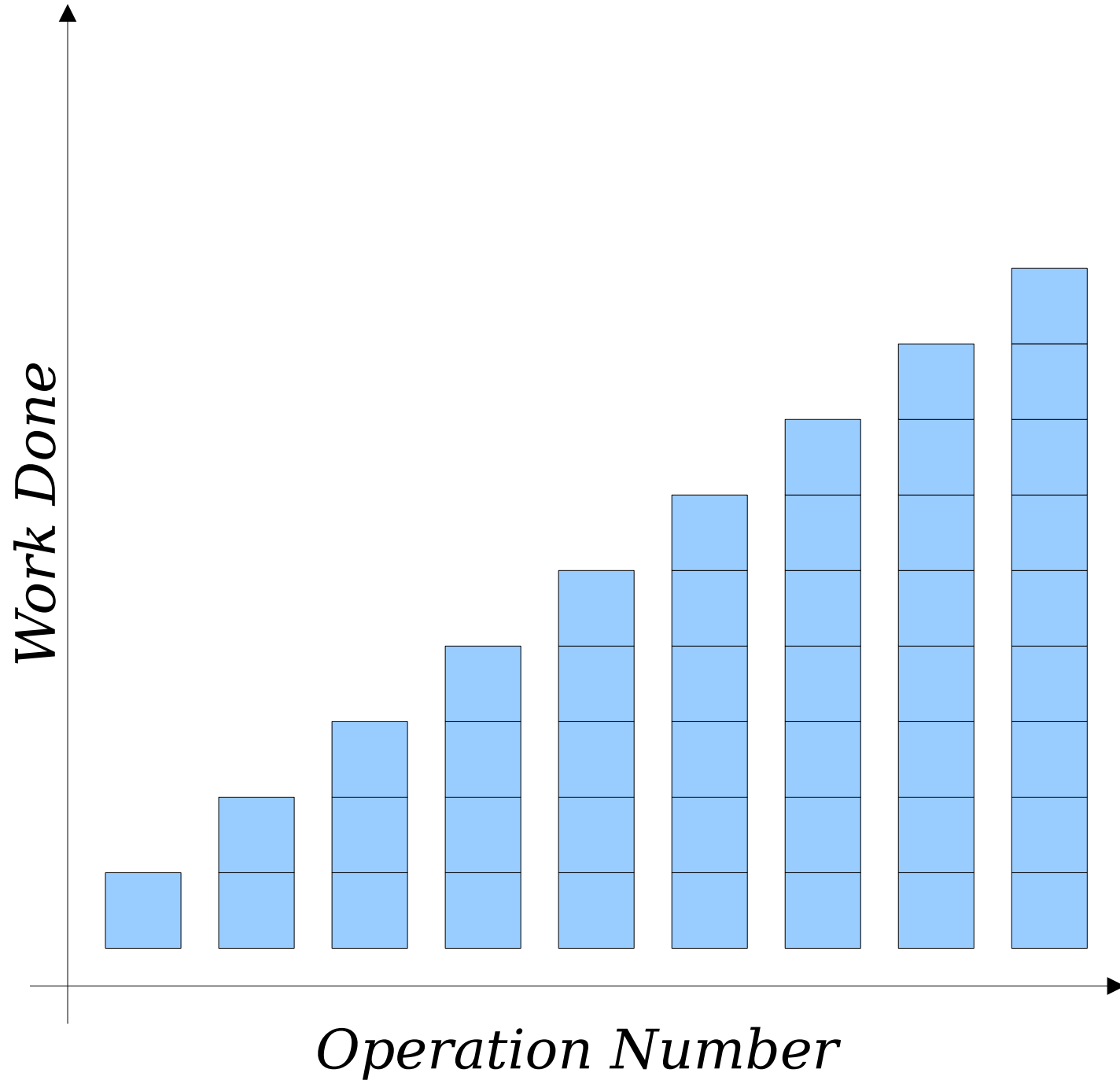
6 Items
Moved

7 Items
Moved



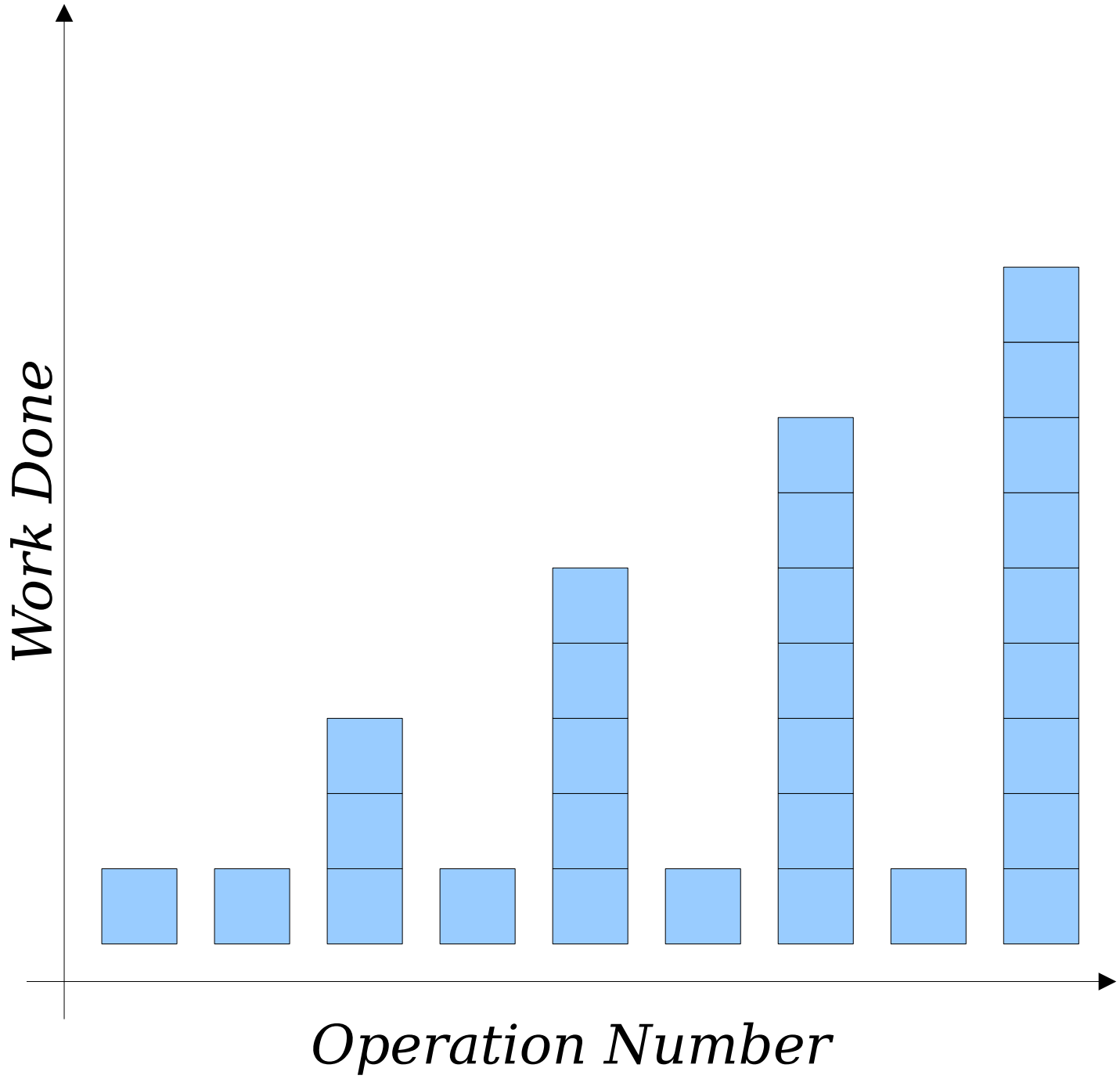


Now, only half the pushes we do will require moving everything to a new array.



Increase array size by ***adding one.***

Increase array size by *adding two*.

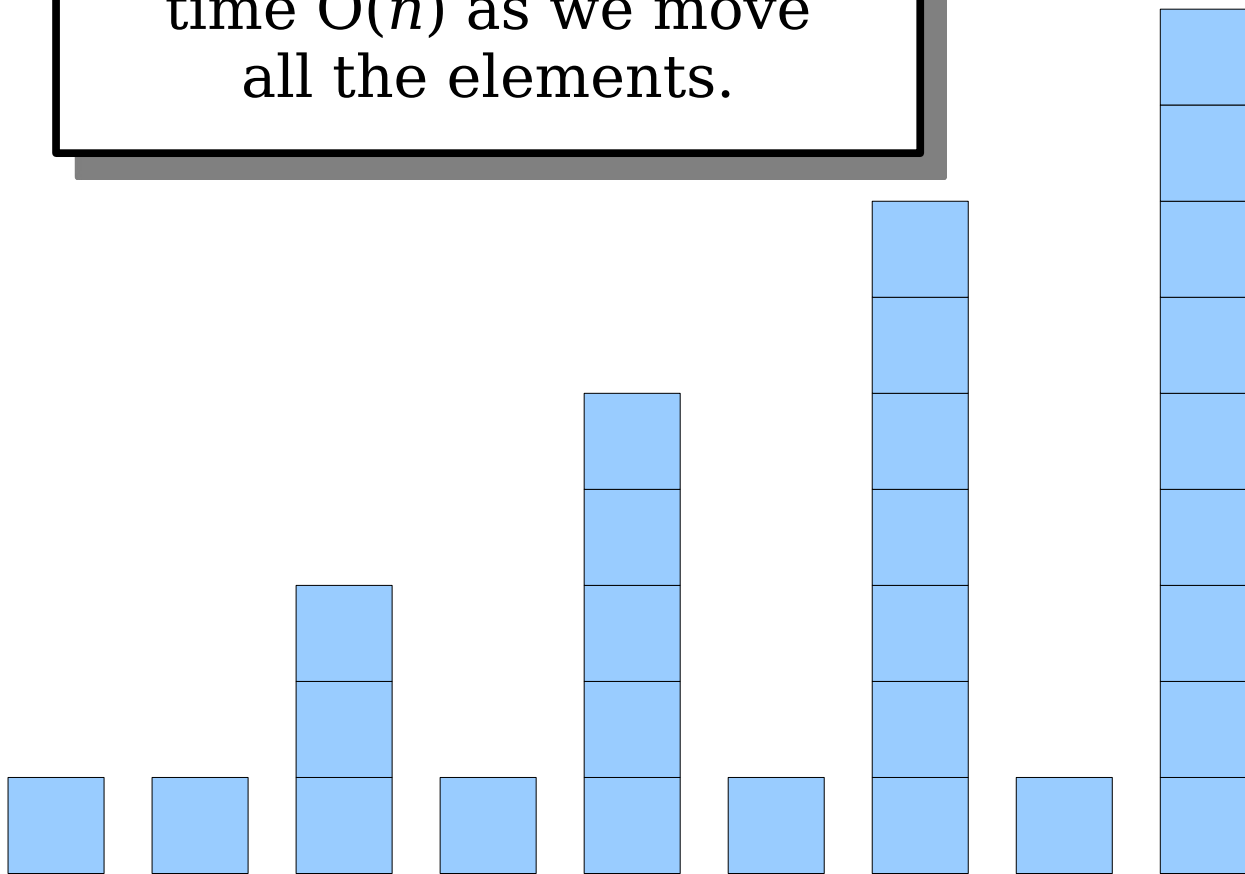


Half of our pushes take time $O(1)$ because there's free space left.

Half of our pushes take time $O(n)$ as we move all the elements.

Increase array size by **adding two**.

Work Done

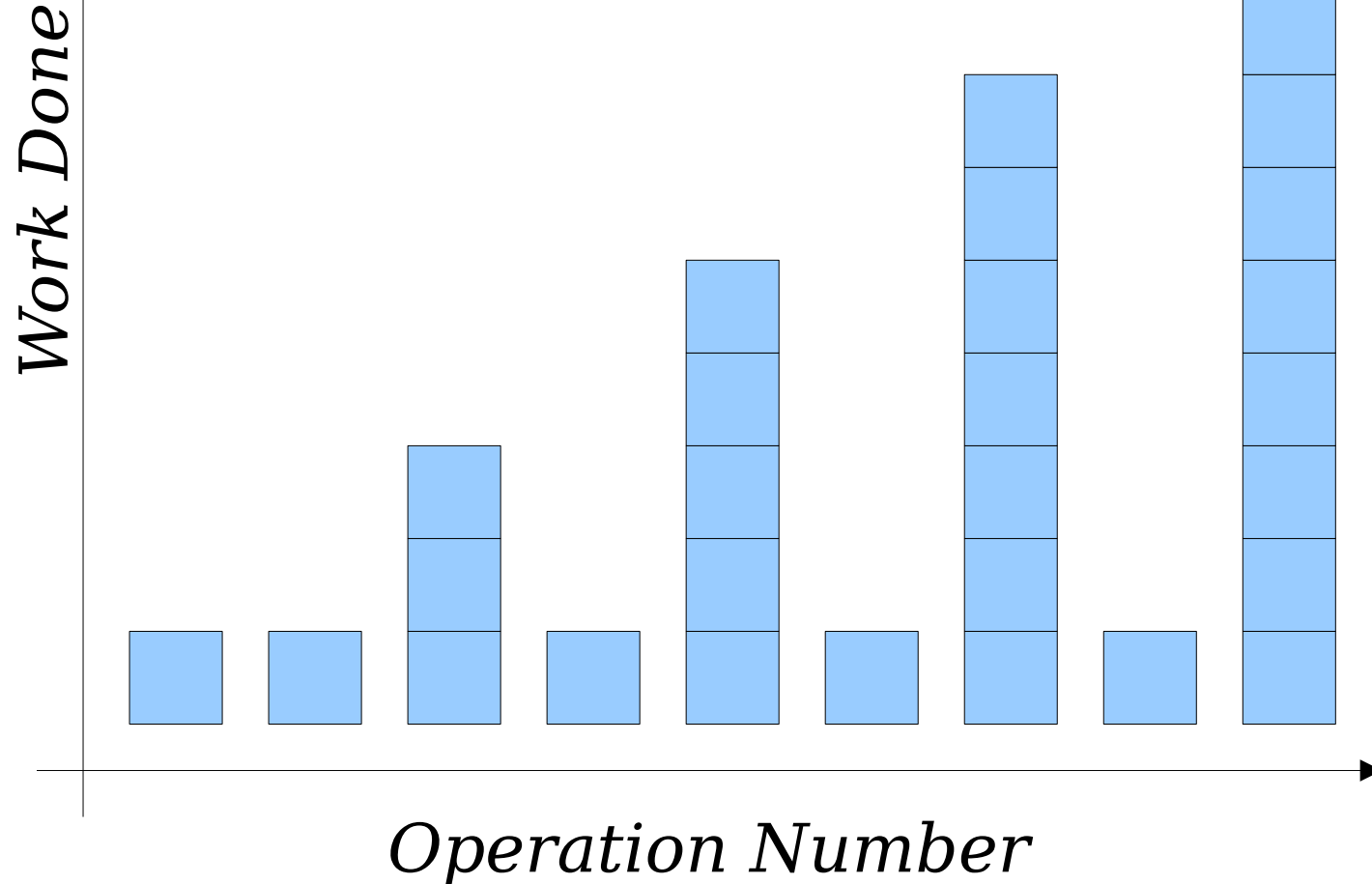


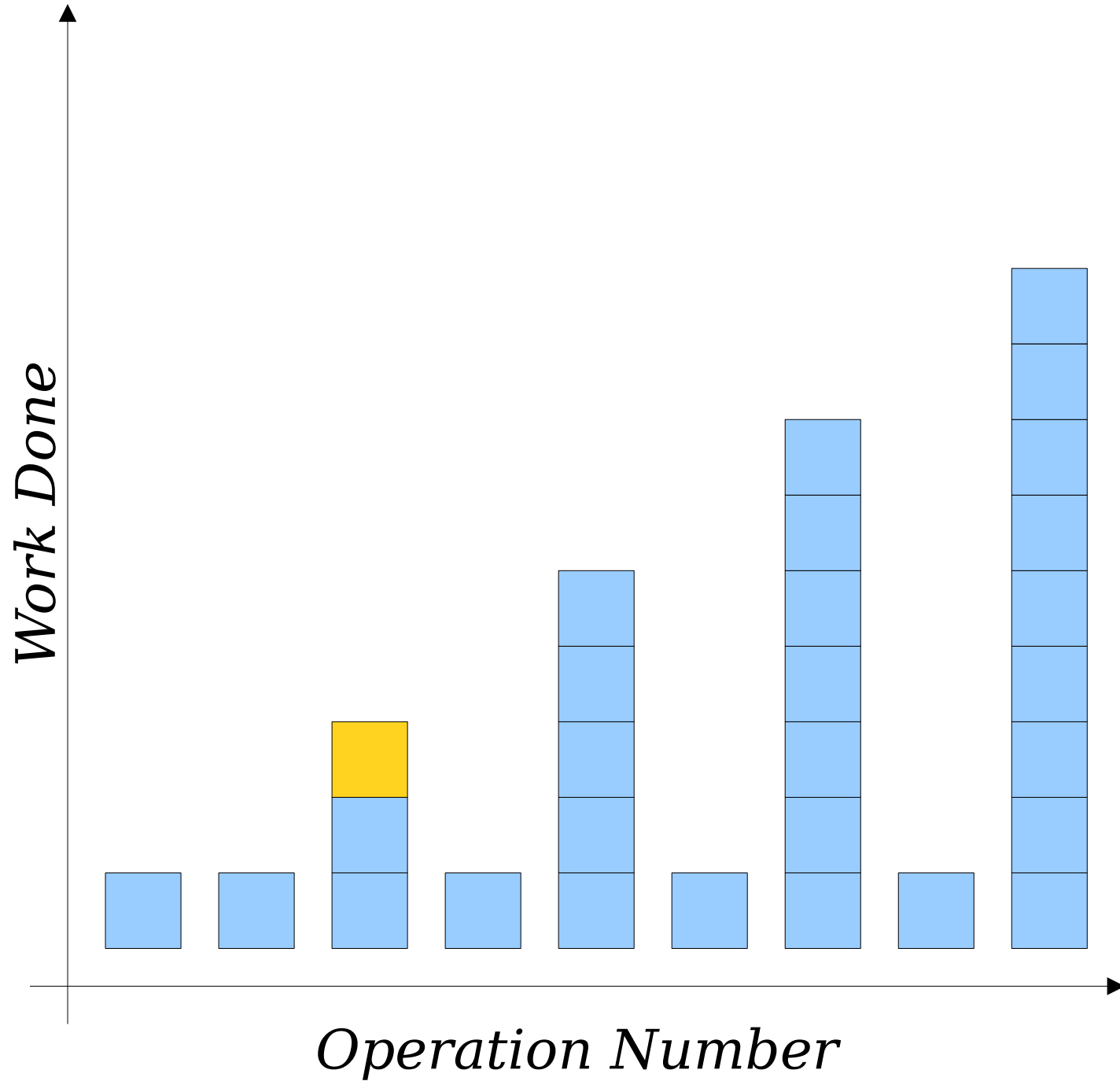
Operation Number

What's the average work done with each push?

To find out, let's see how much total work was done.

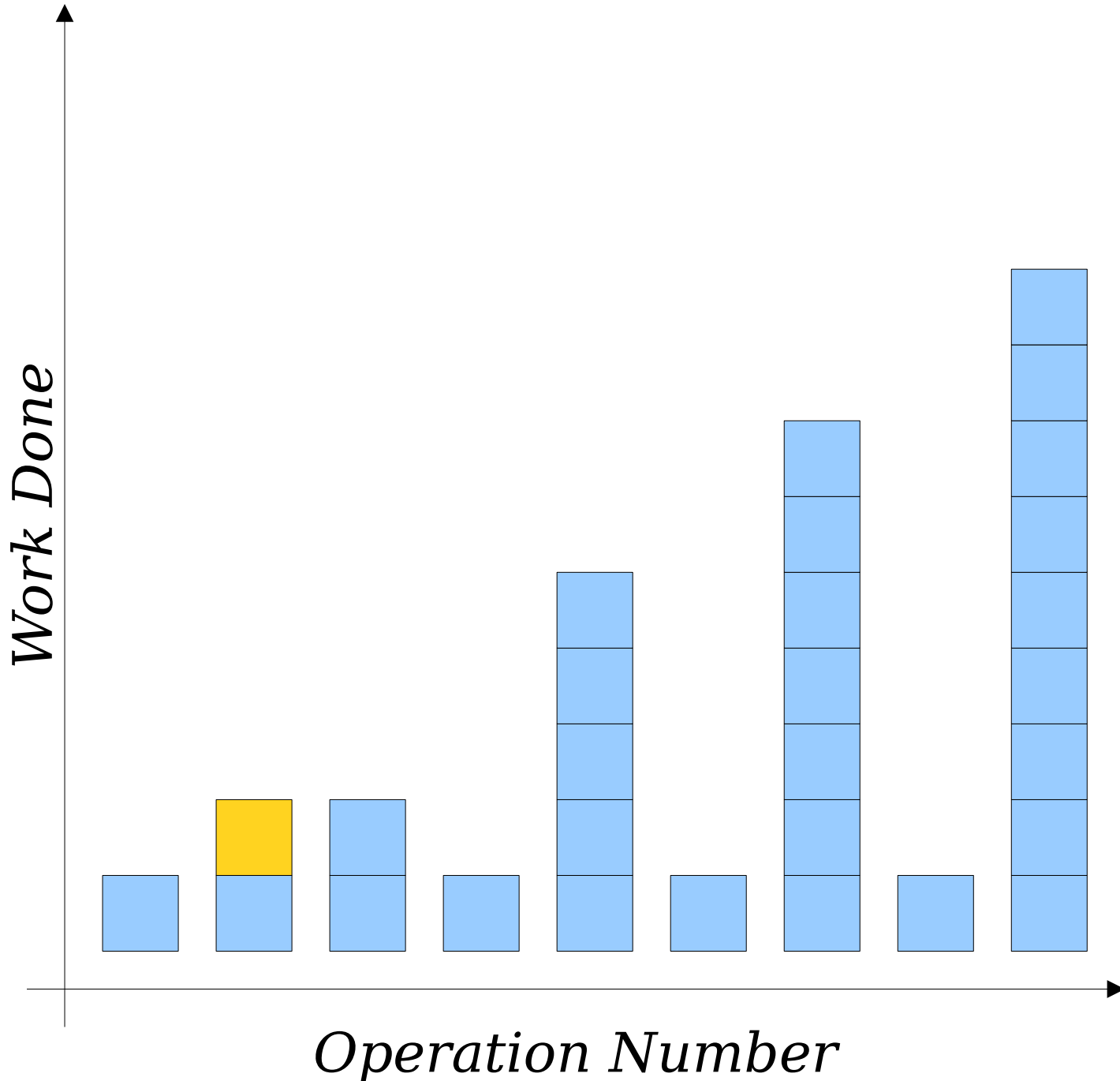
Increase array size by **adding two**.

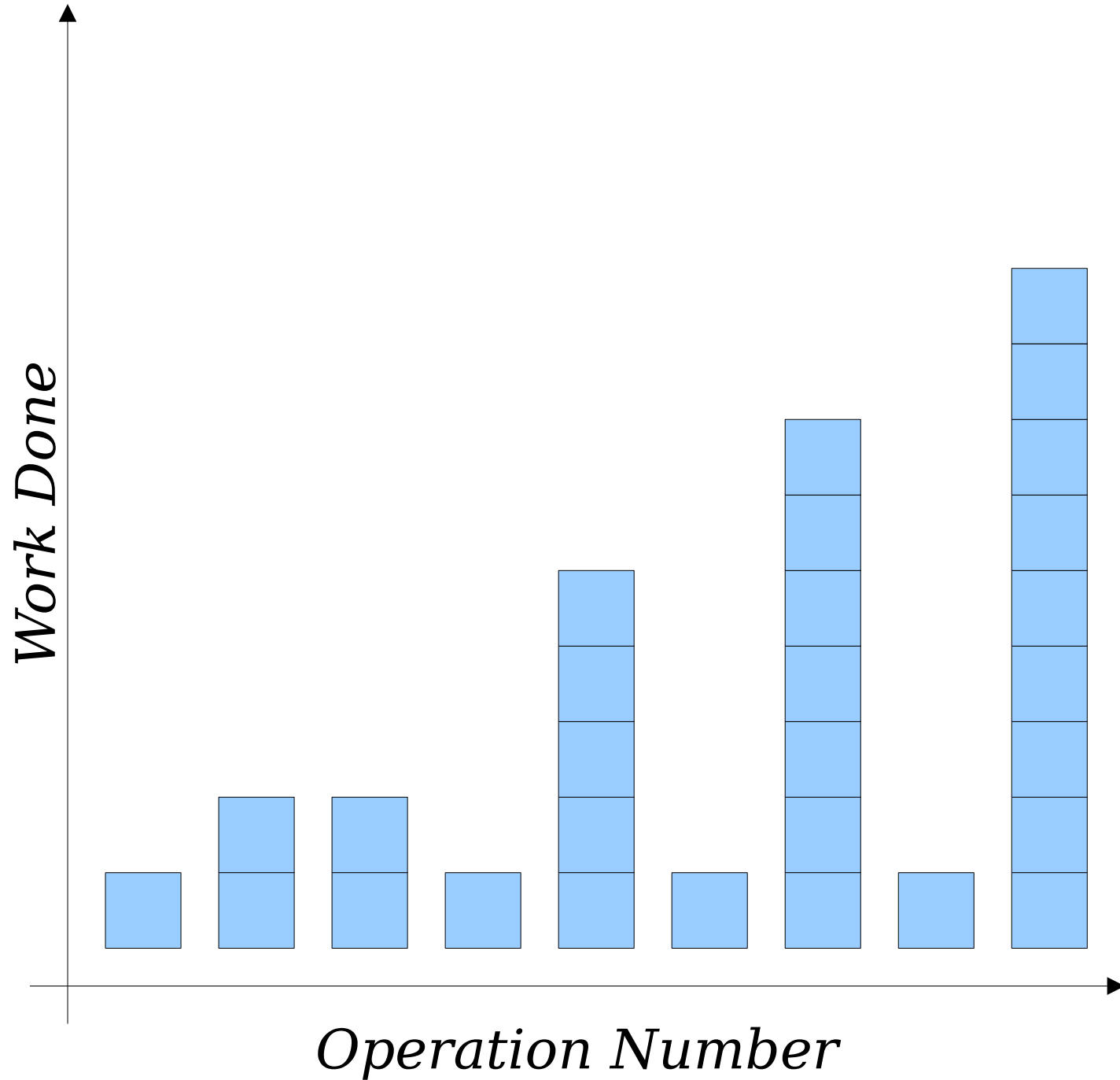




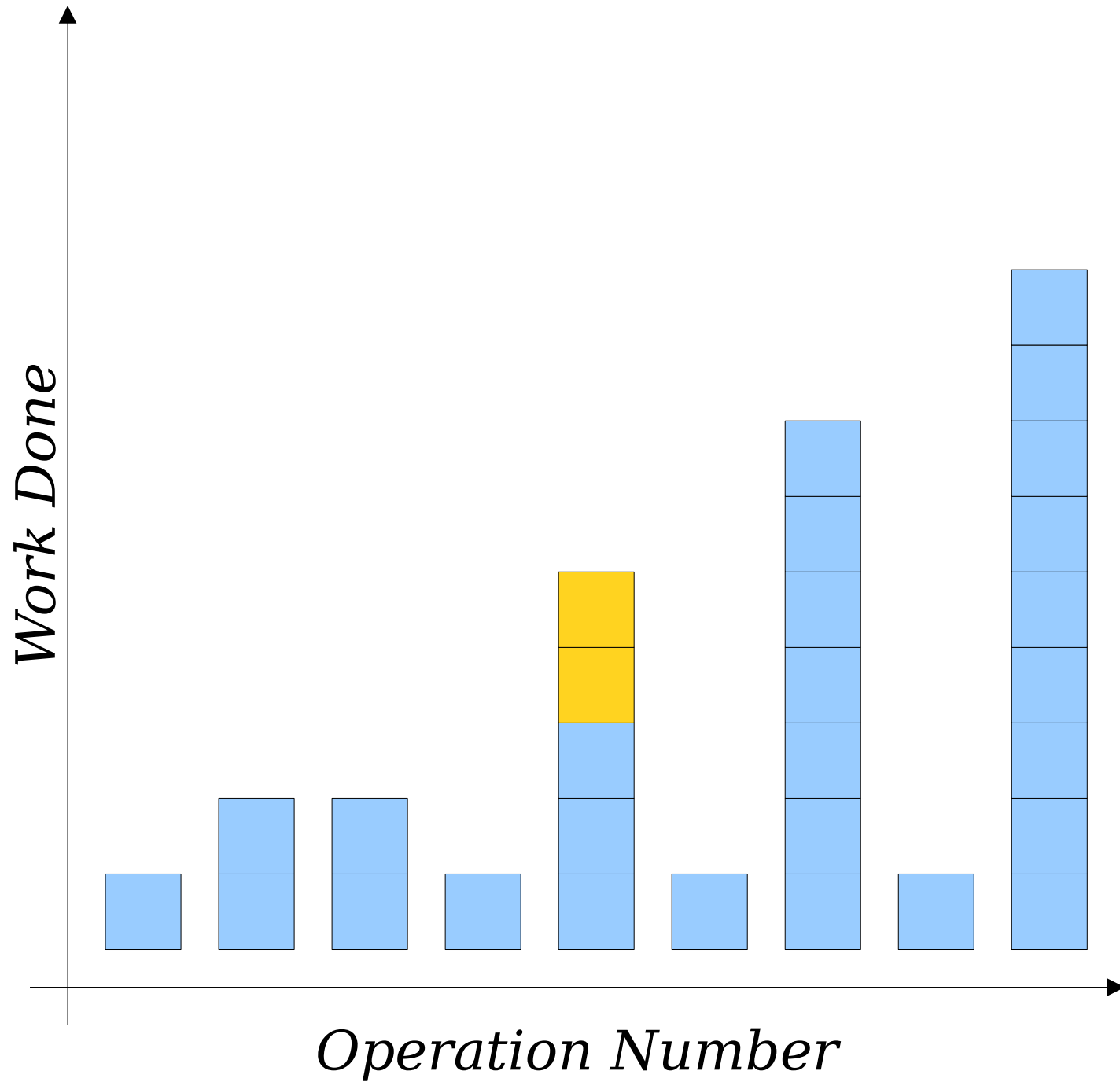
Increase array size by *adding two*.

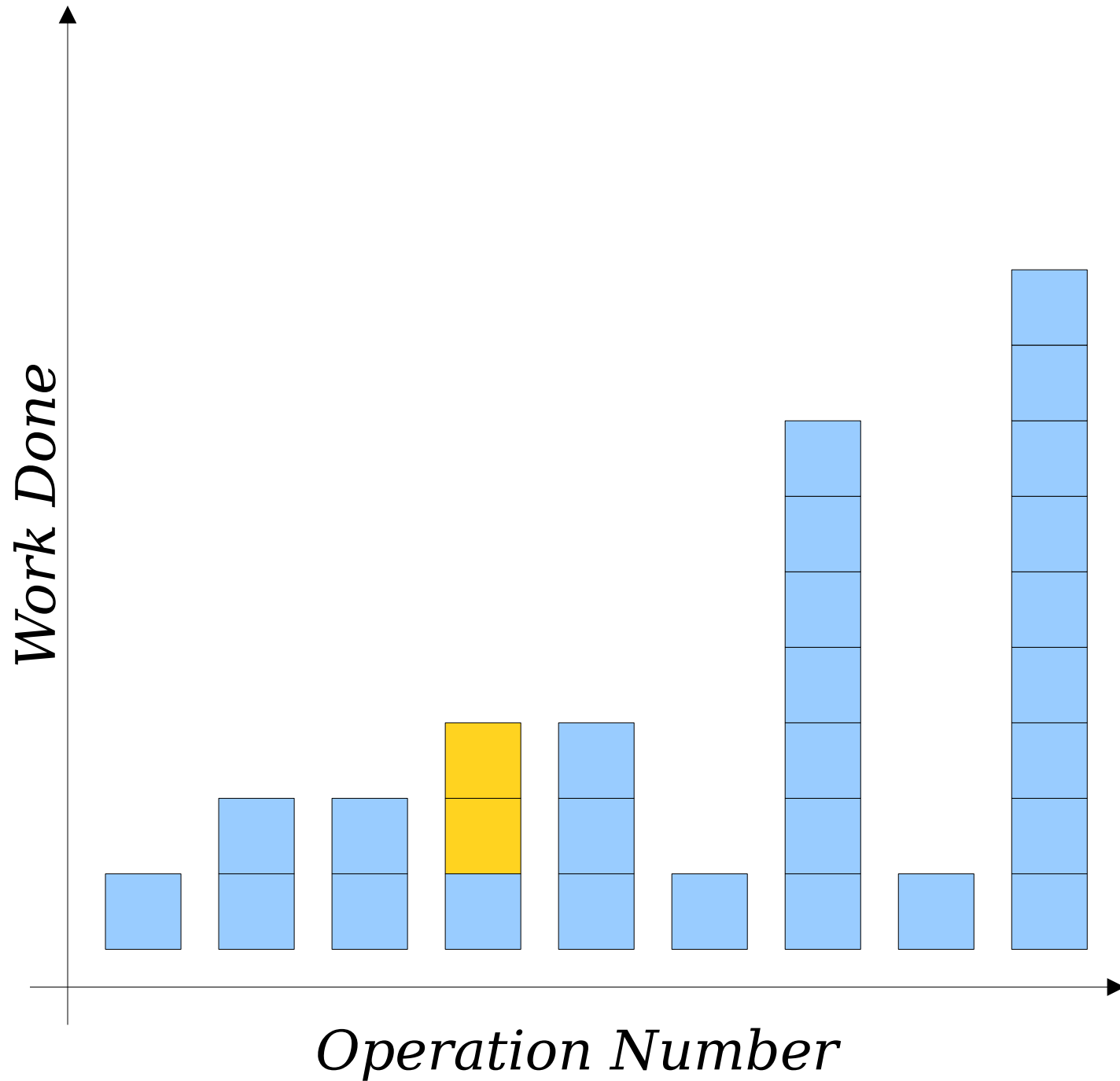
Increase array size by *adding two*.



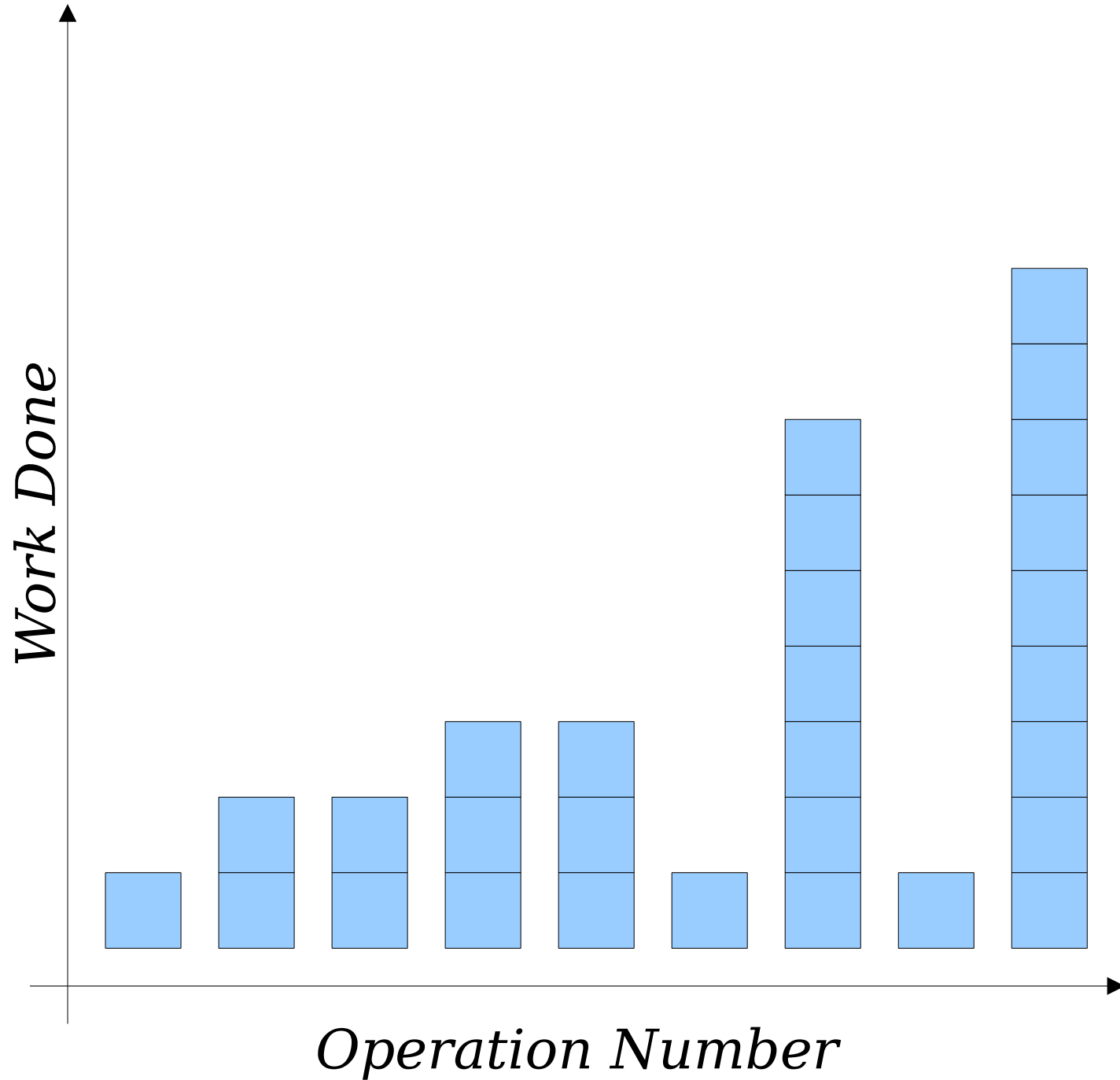


Increase array size by *adding two*.

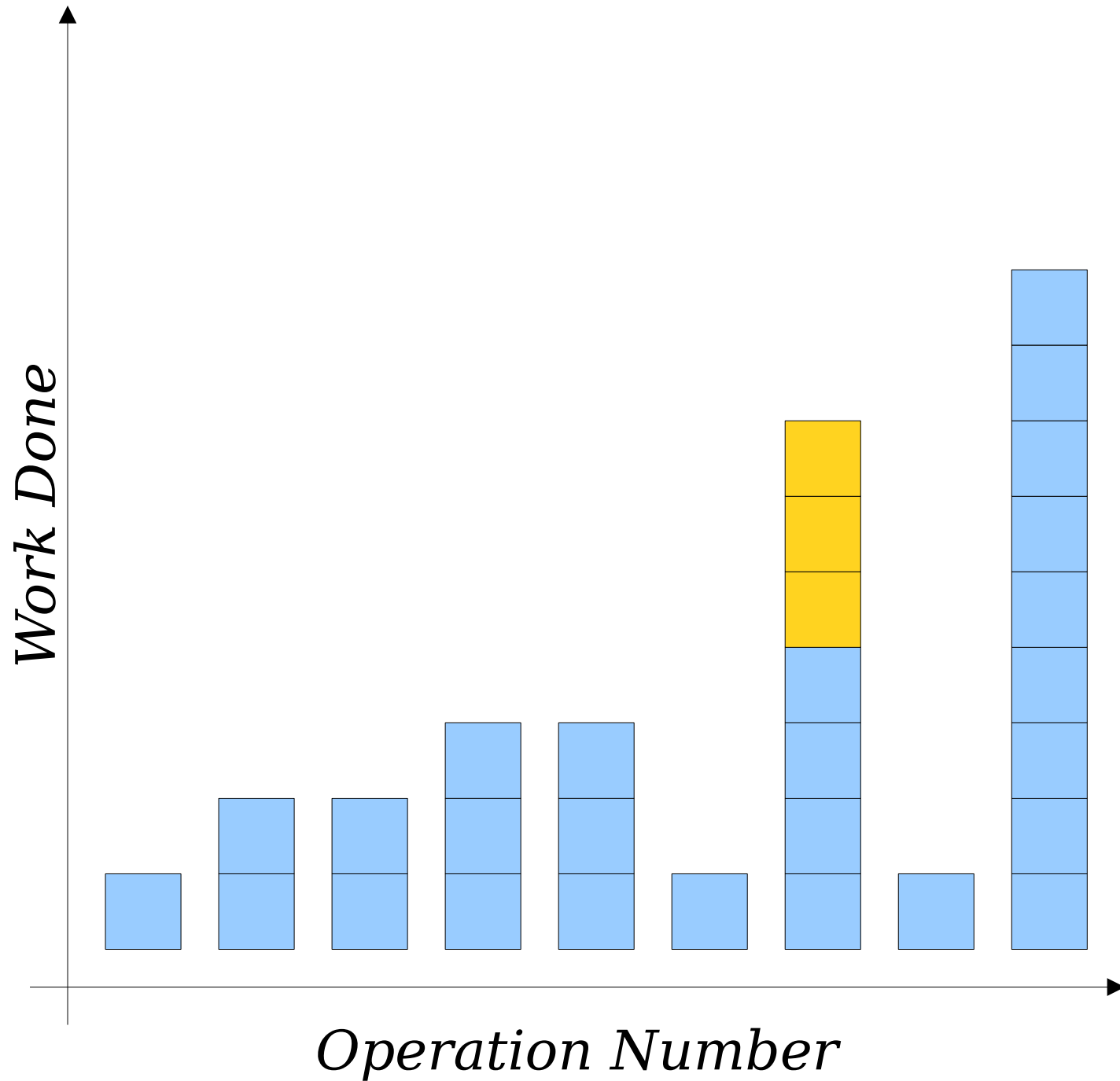




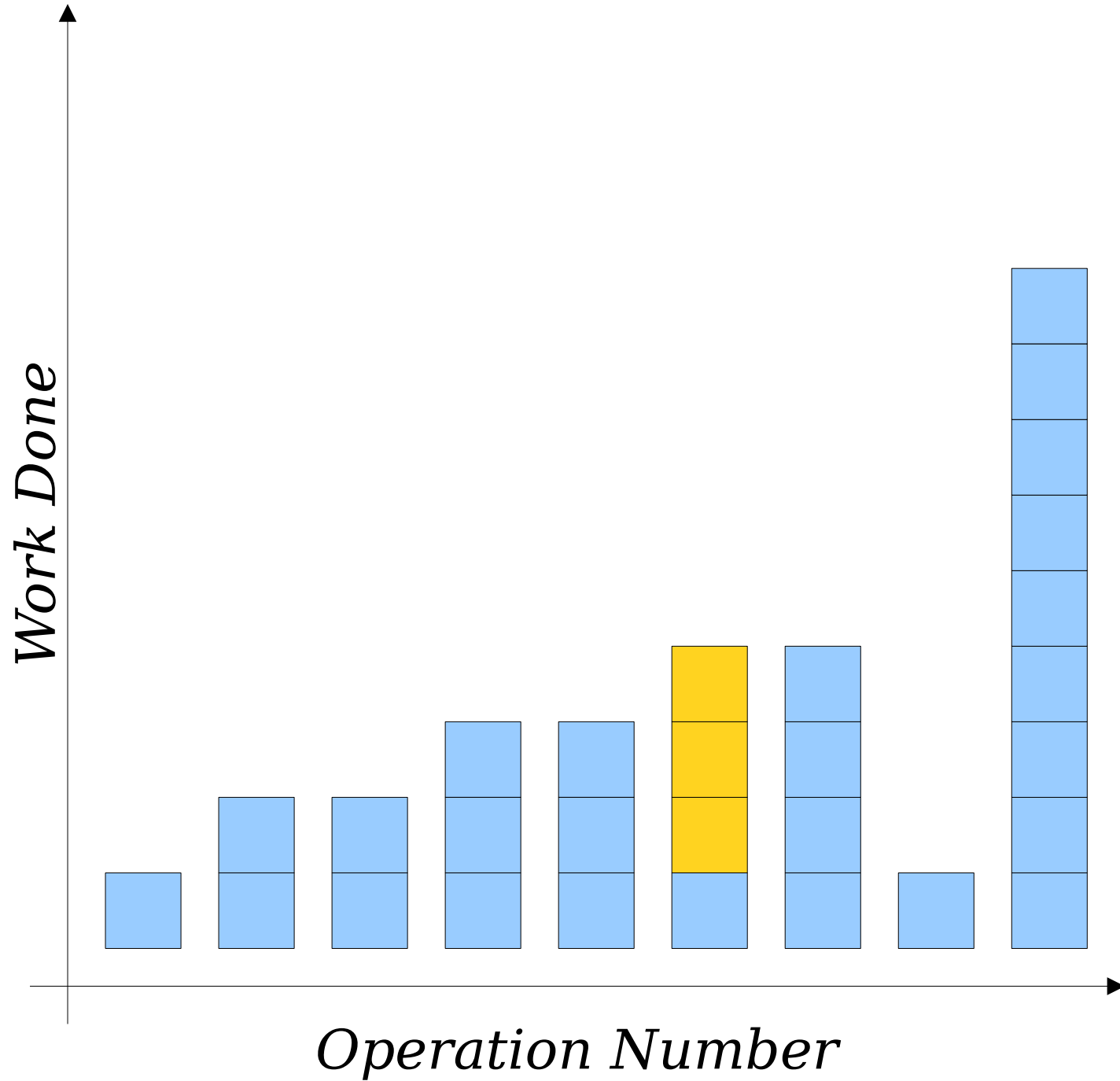
Increase array size by *adding two*.



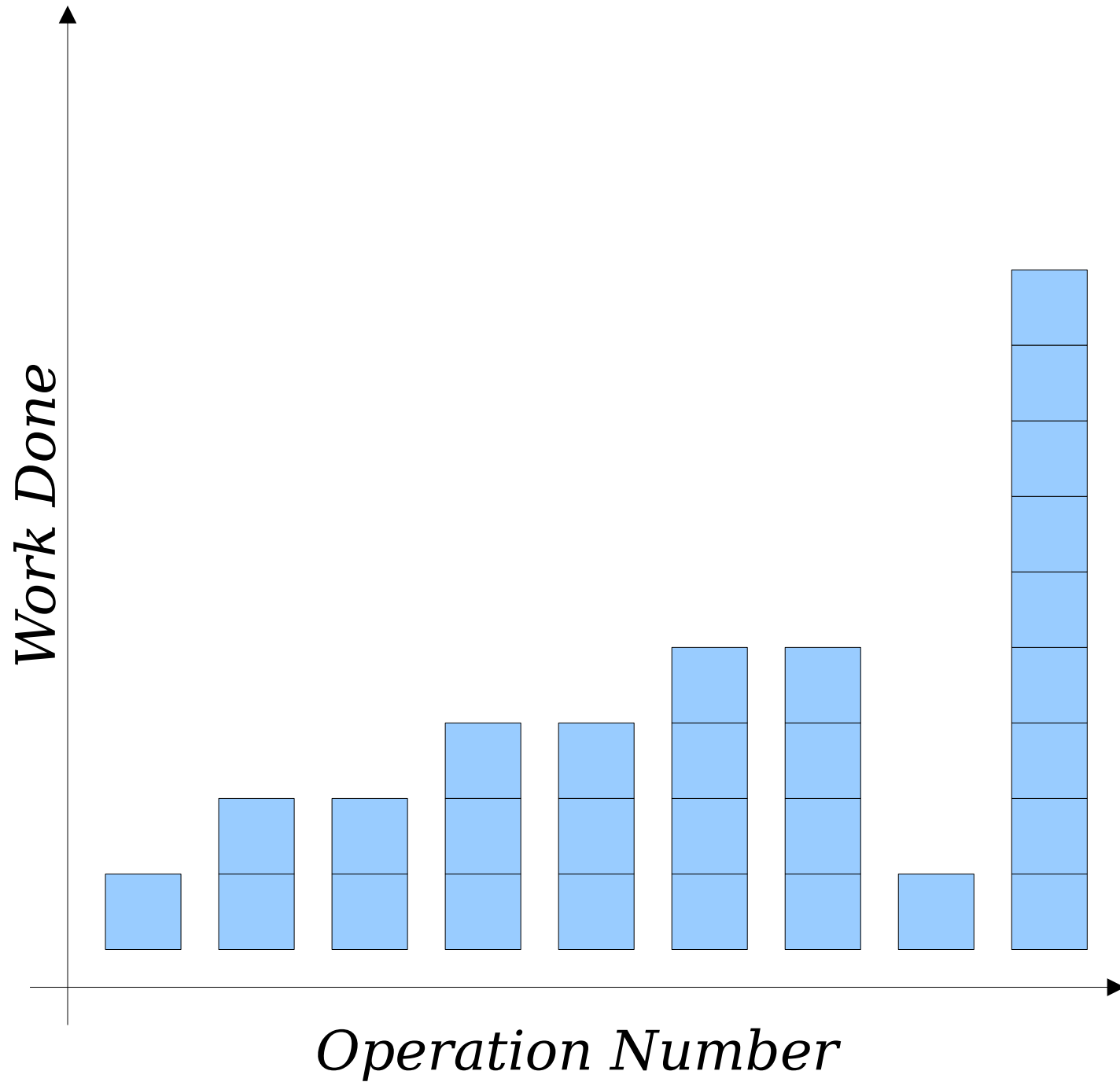
Increase array size by *adding two*.



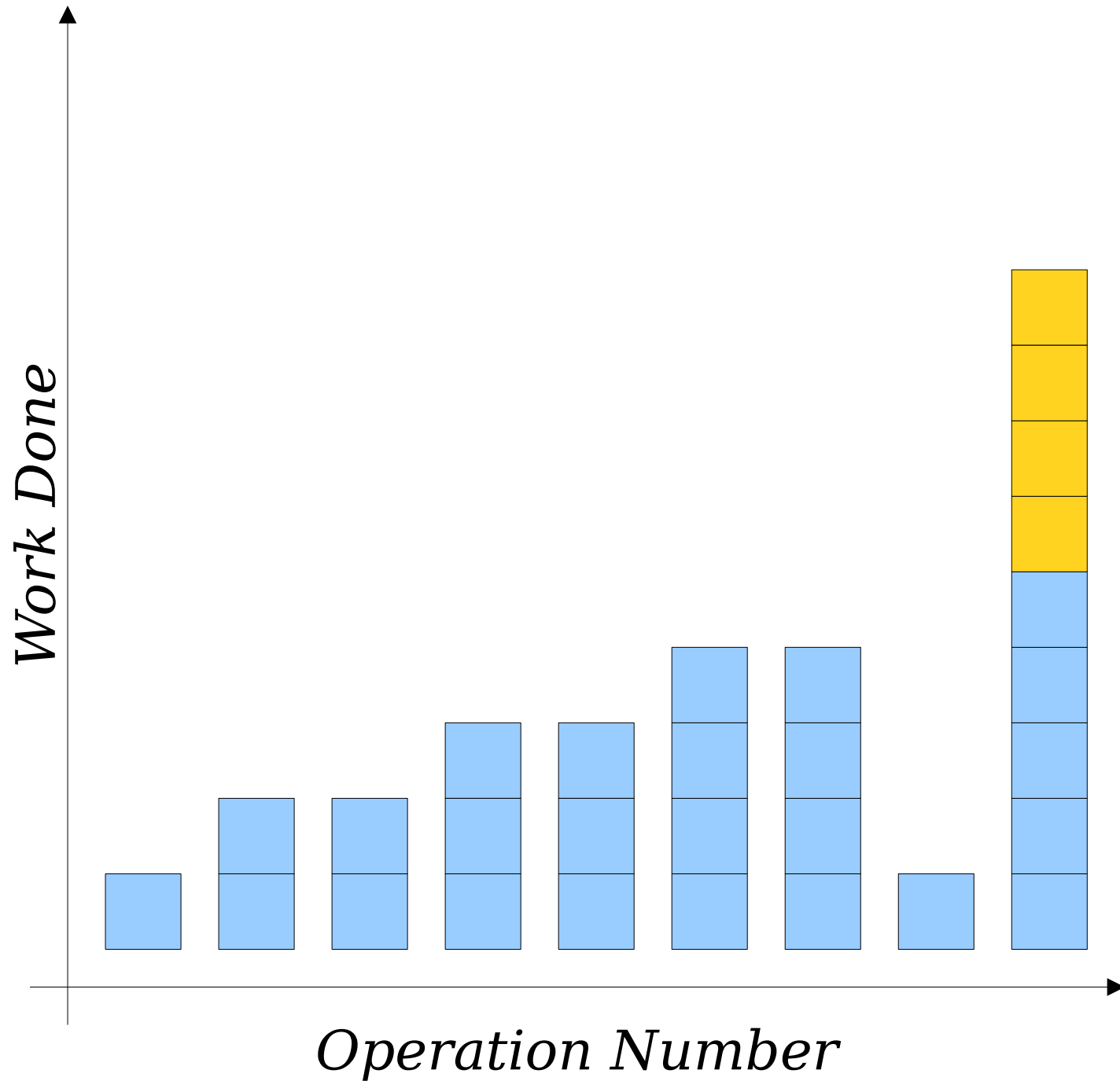
Increase array size by *adding two*.



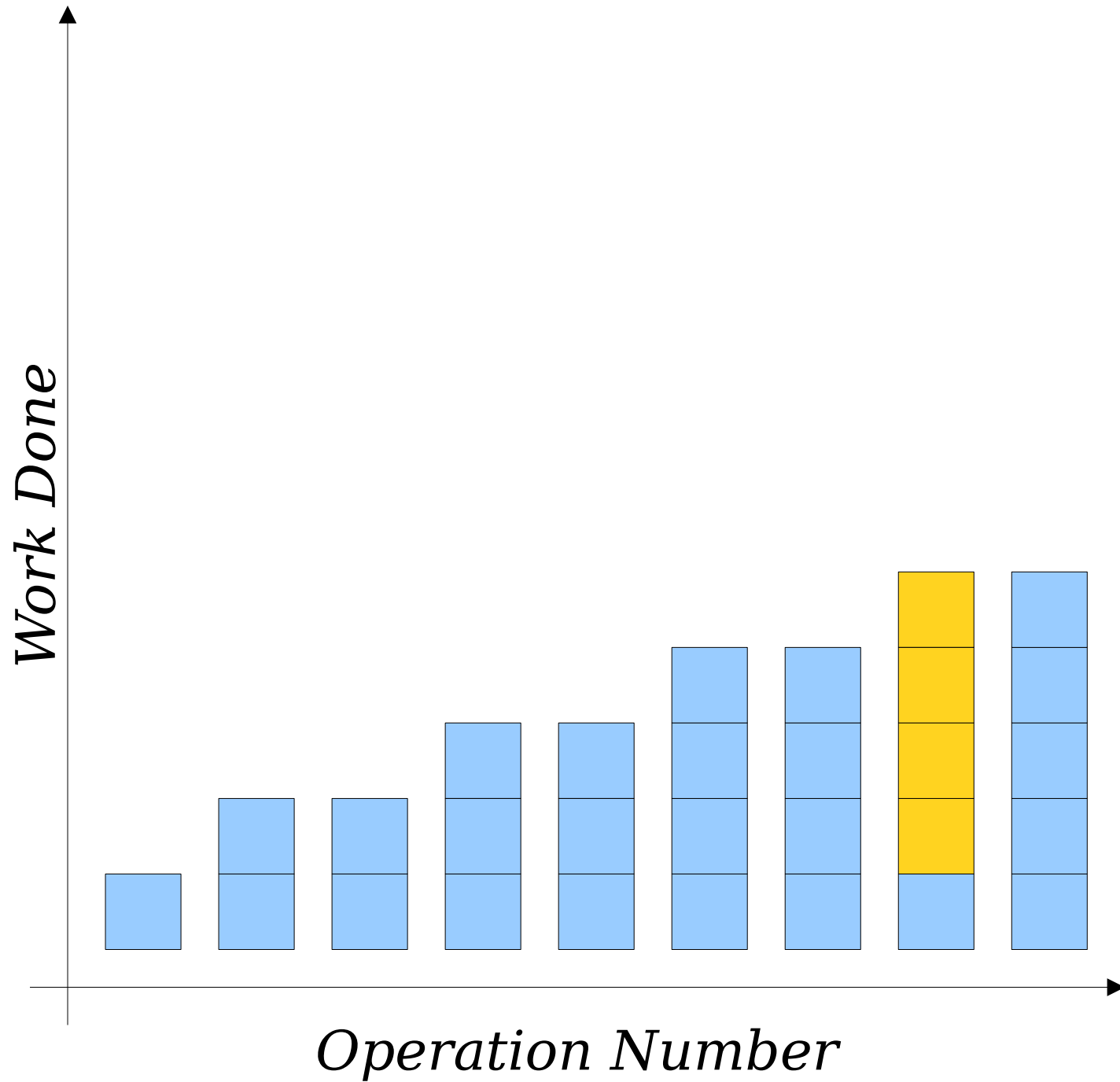
Increase array size by *adding* *two*.



Increase array size by *adding two*.

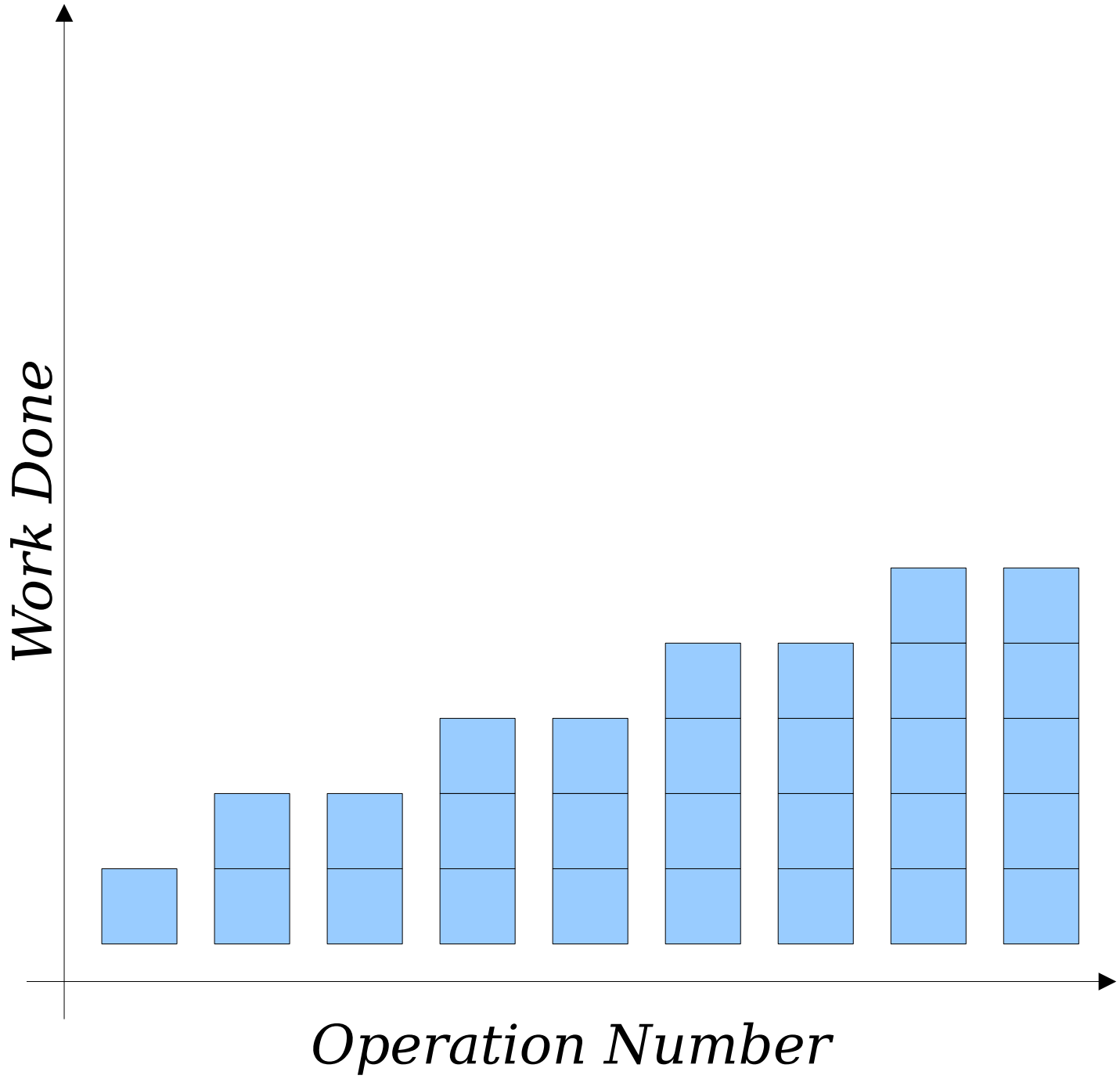


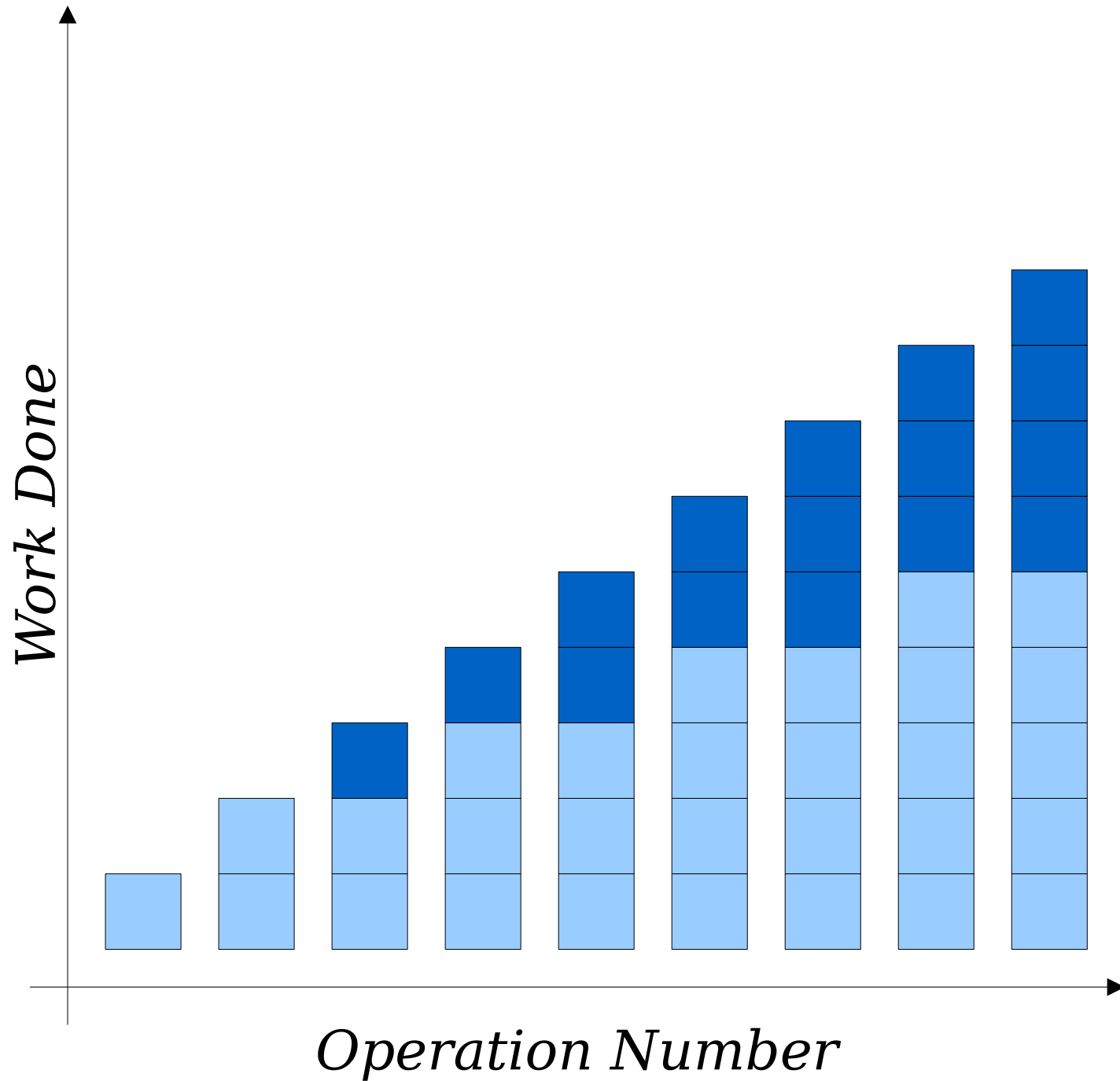
Increase array size by *adding two*.



Increase array size by ***adding two.***

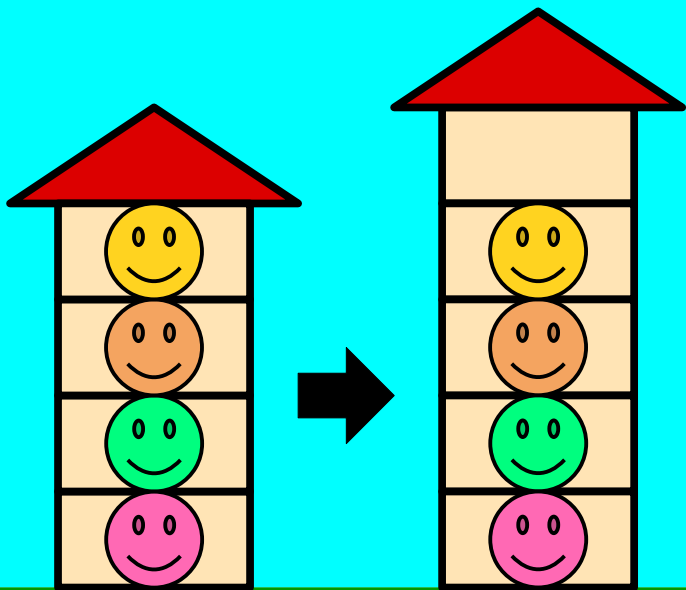
Increase array size by *adding two*.

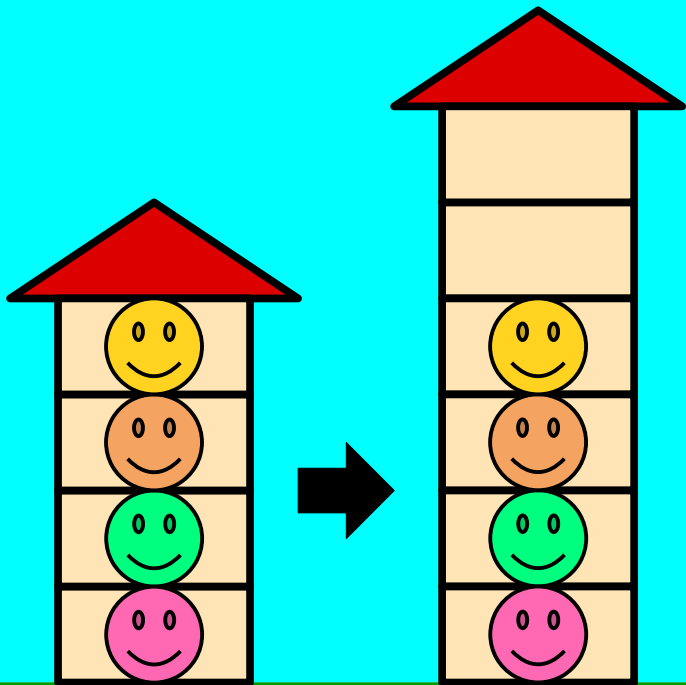


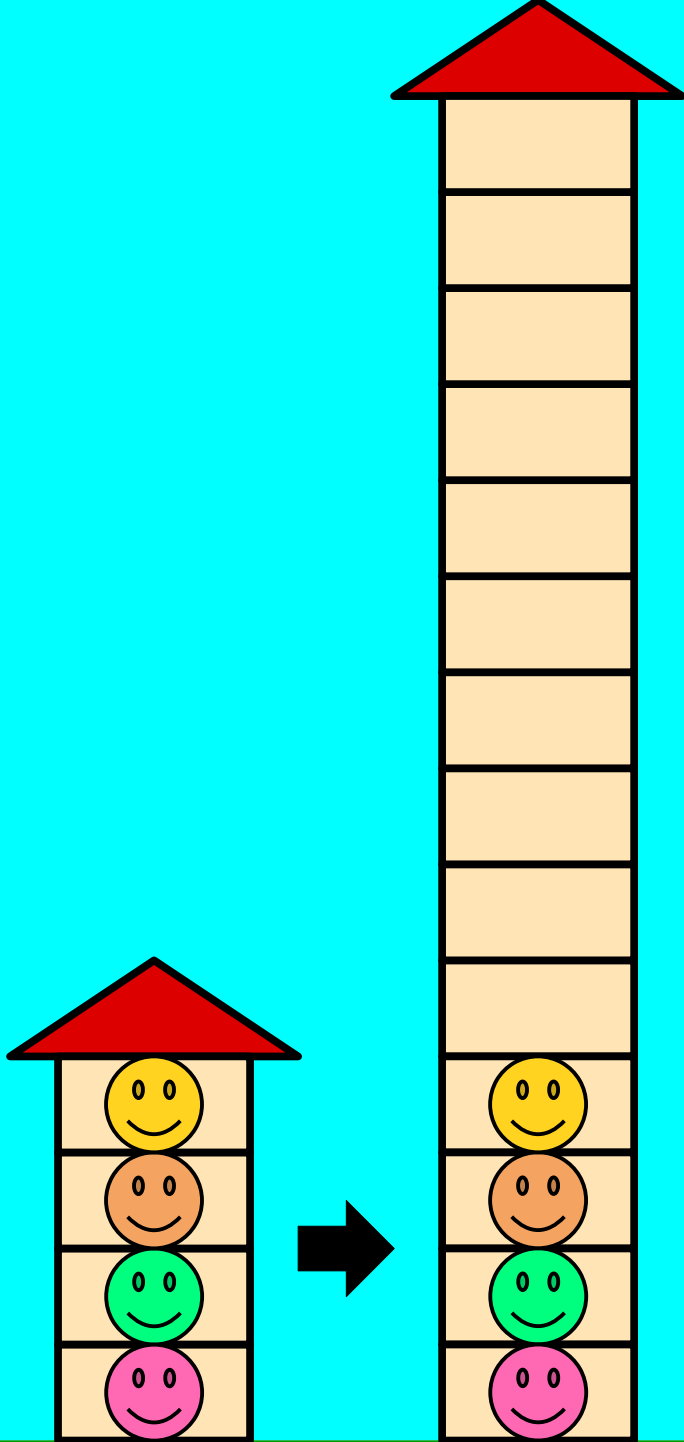


Increase array size by *adding two*.

This roughly halves the work done.

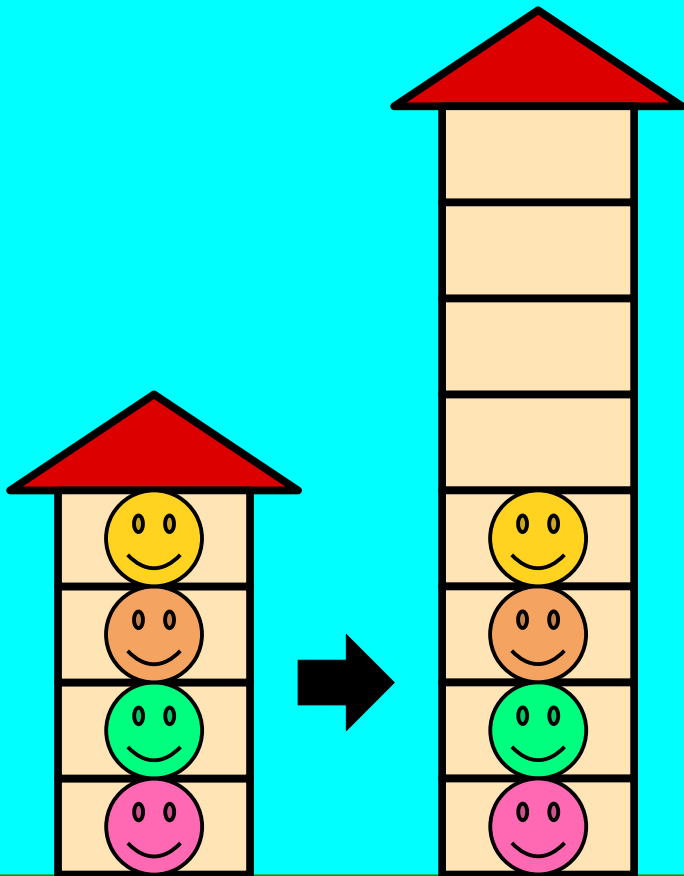






If we make the new array too big, we're might not make use of all the new space.

What's a good compromise?



Idea: Make the new array twice as big as the old one.

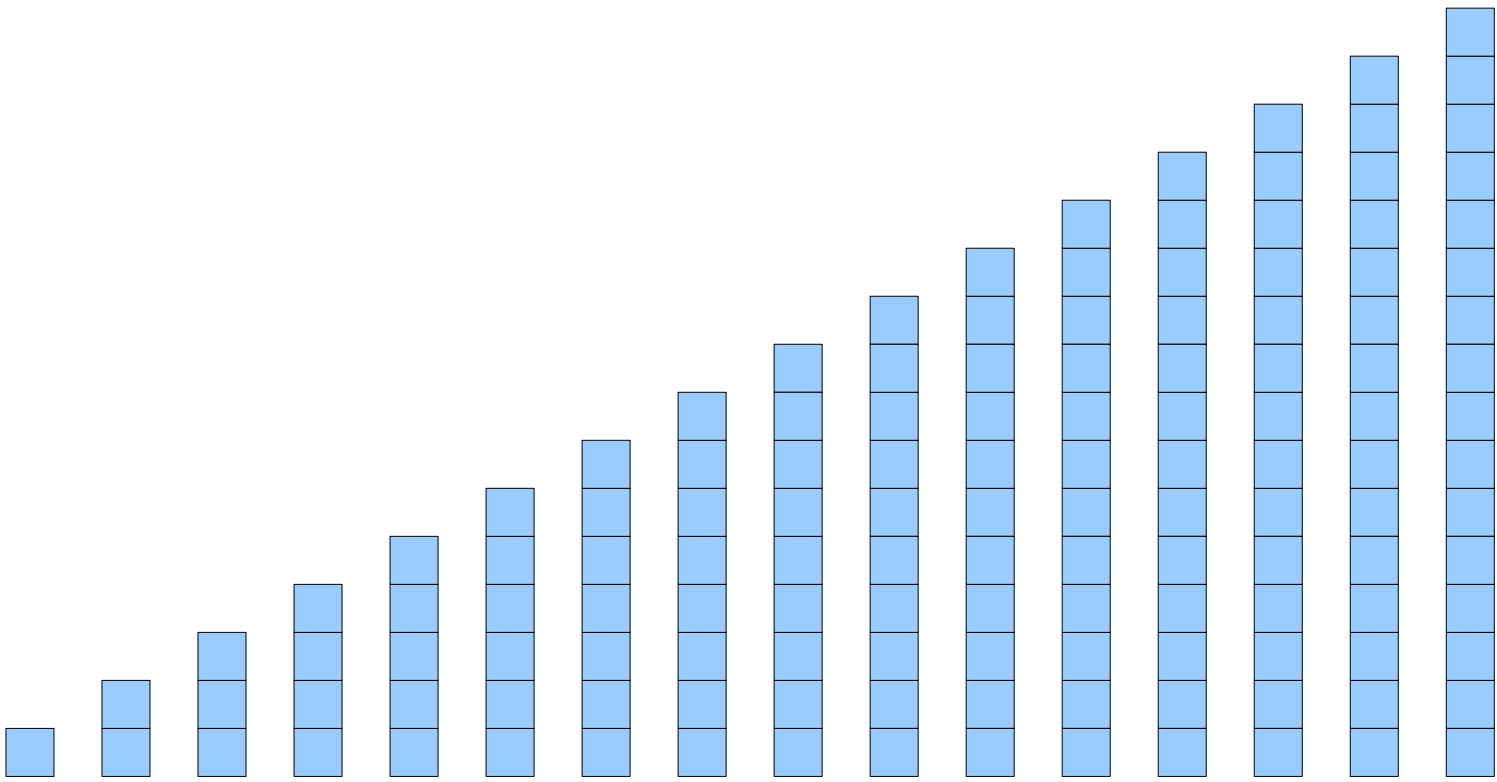
This gives us a lot of free space, and we never use more than twice the space we need.

How do we analyze this?

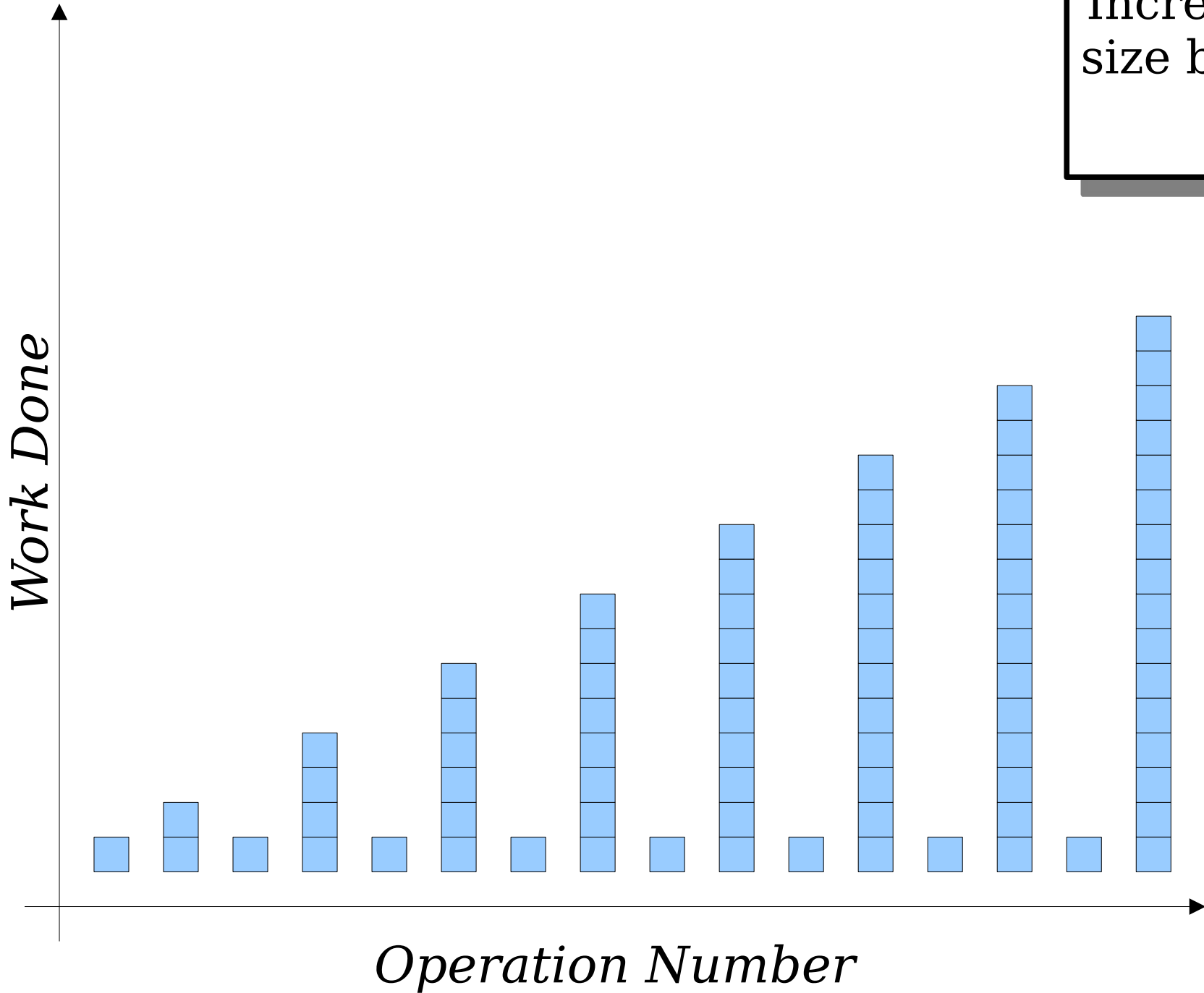
Work Done

Operation Number

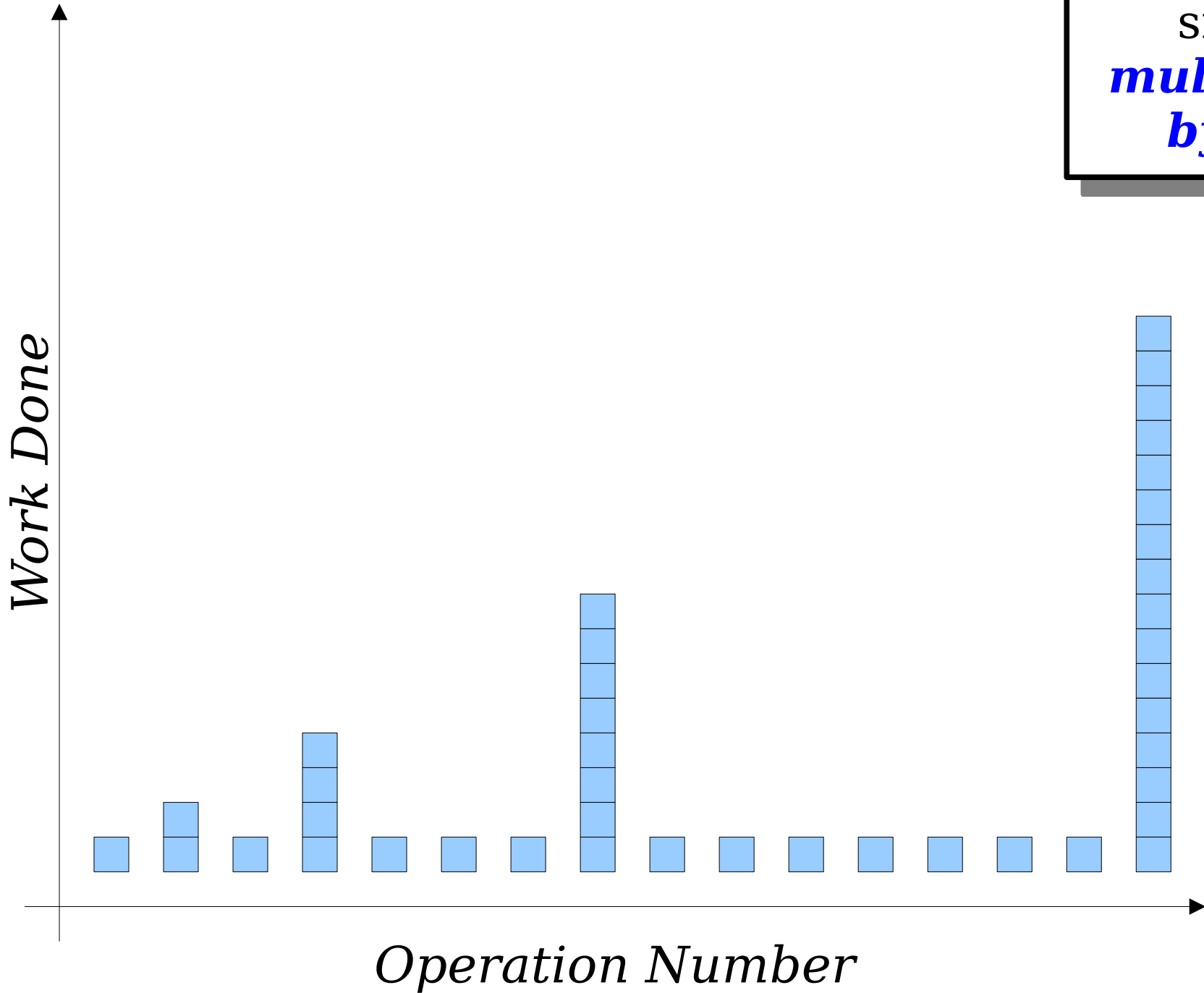
Increase array size by ***adding one.***

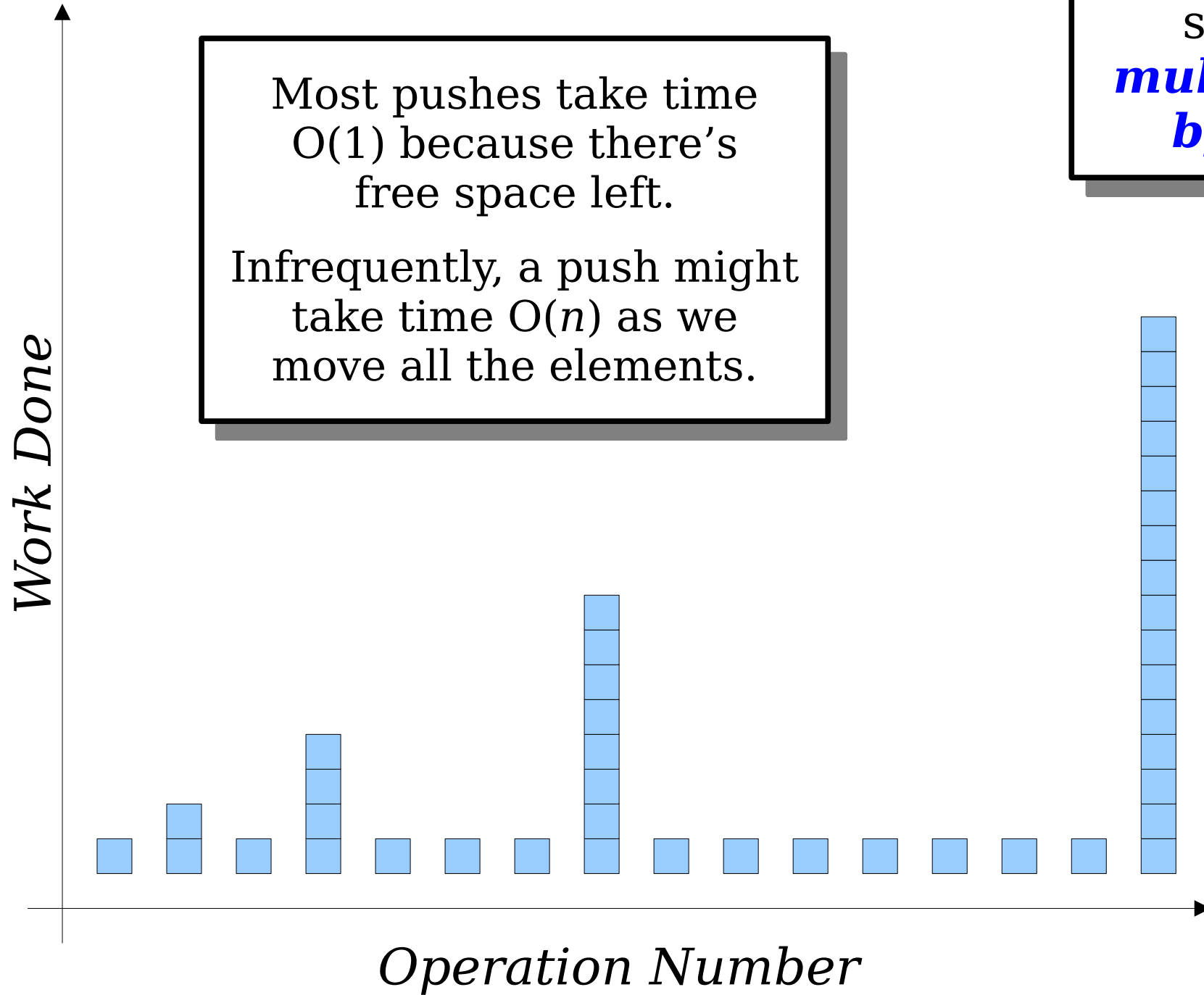


Increase array size by *adding two*.



Increase array size by
size by
multiplying
by two.





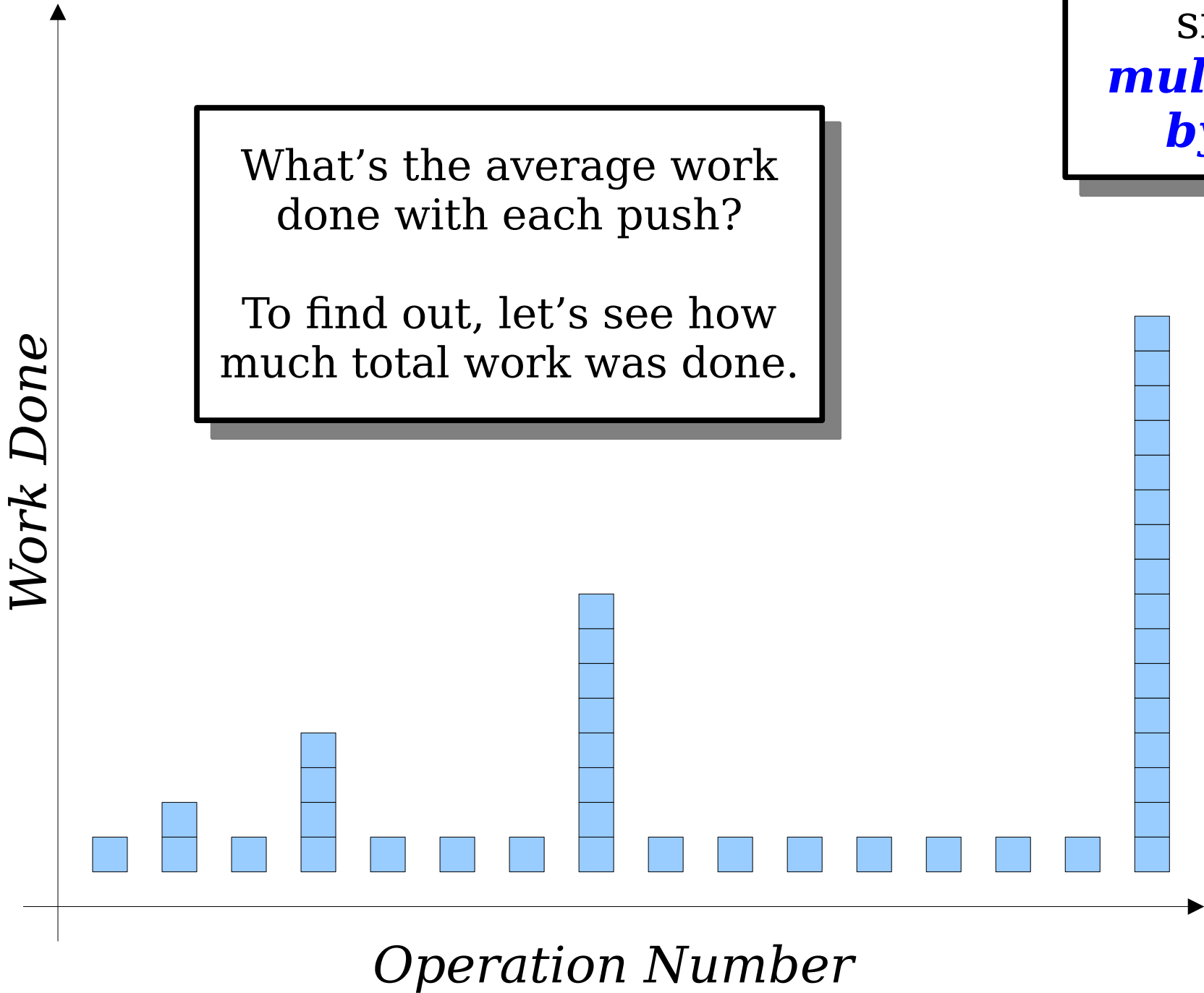
Most pushes take time $O(1)$ because there's free space left.

Infrequently, a push might take time $O(n)$ as we move all the elements.

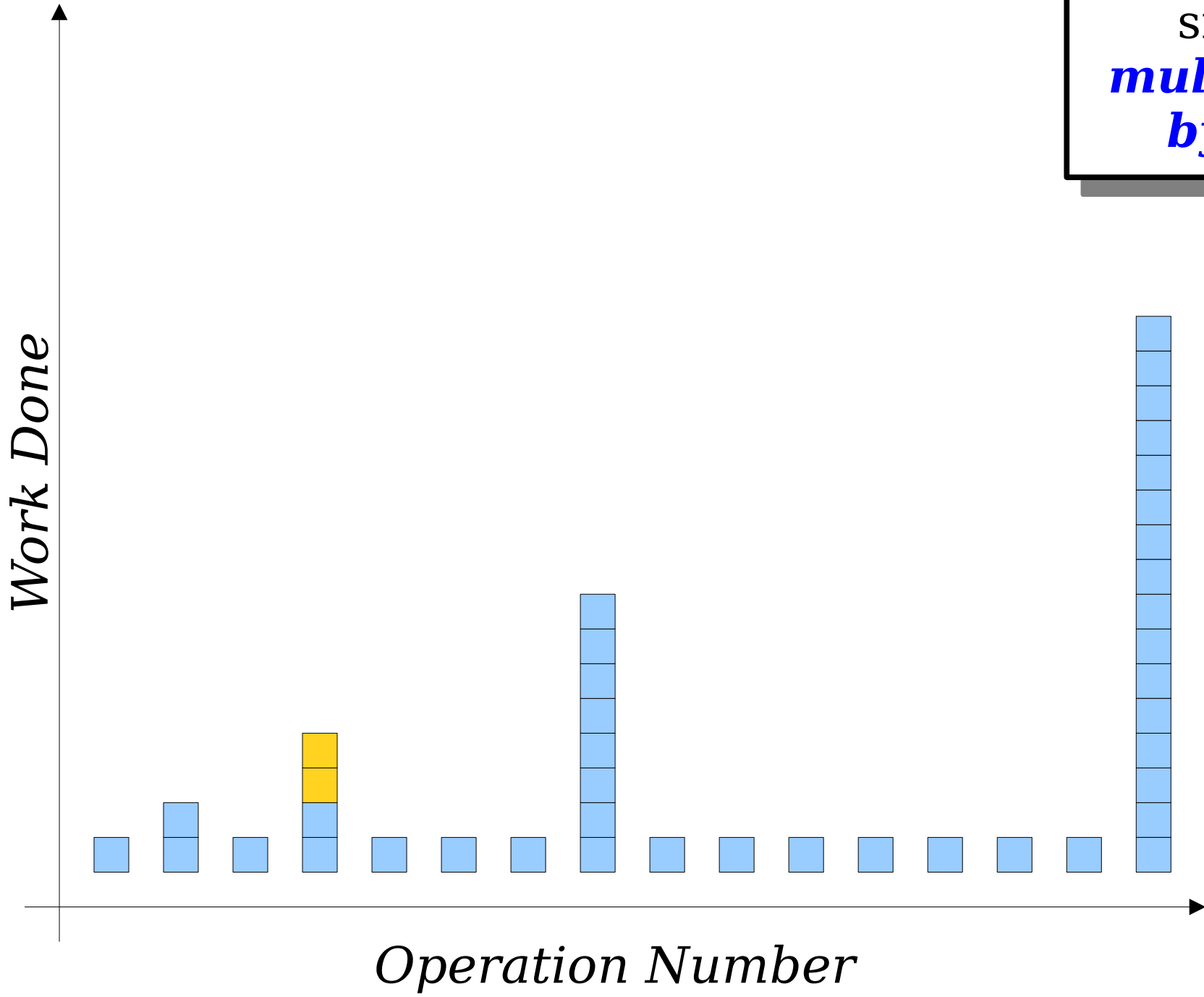
Increase array size by ***multiplying by two.***

Increase array size by *multiplying by two*.

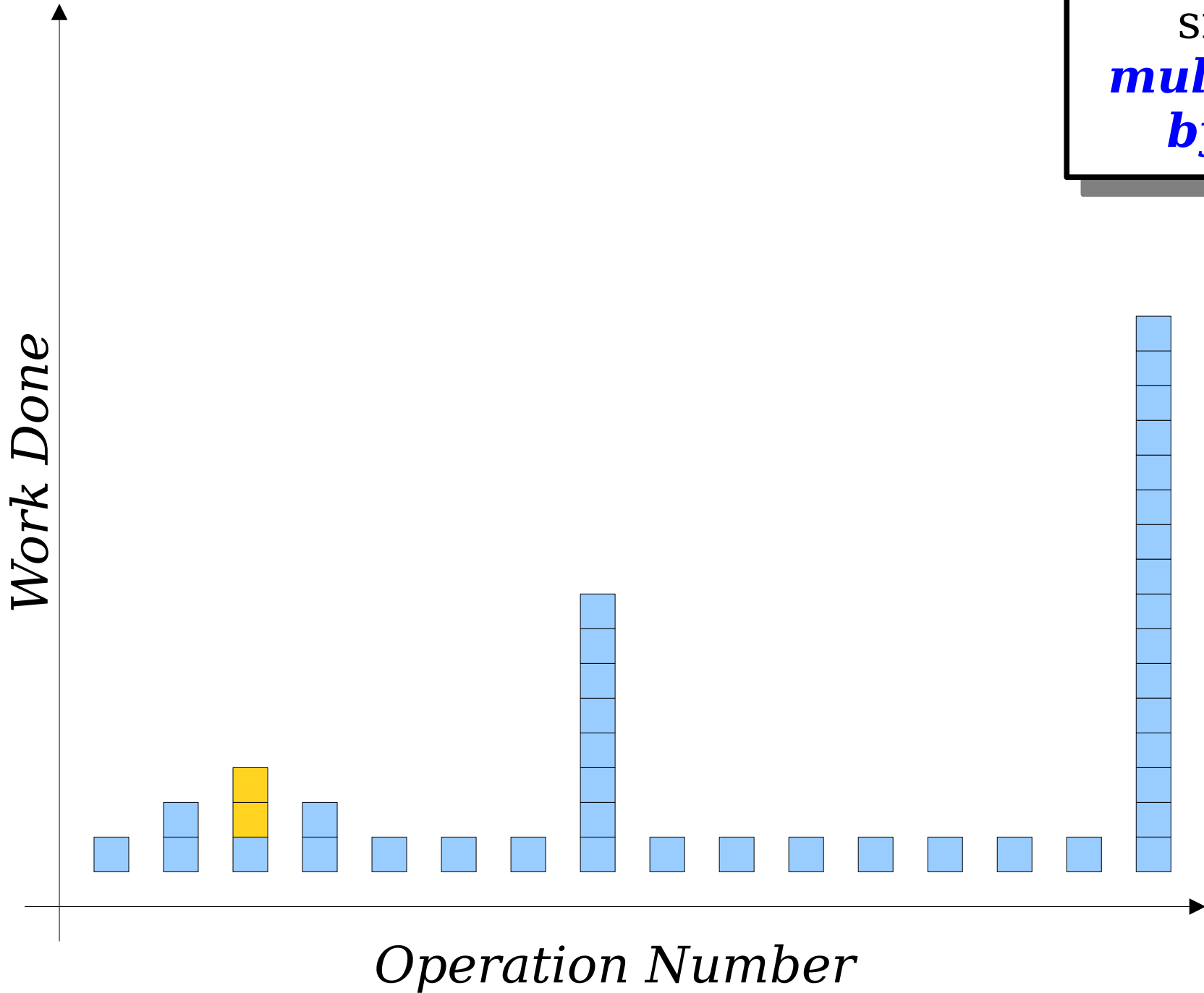
What's the average work done with each push?
To find out, let's see how much total work was done.



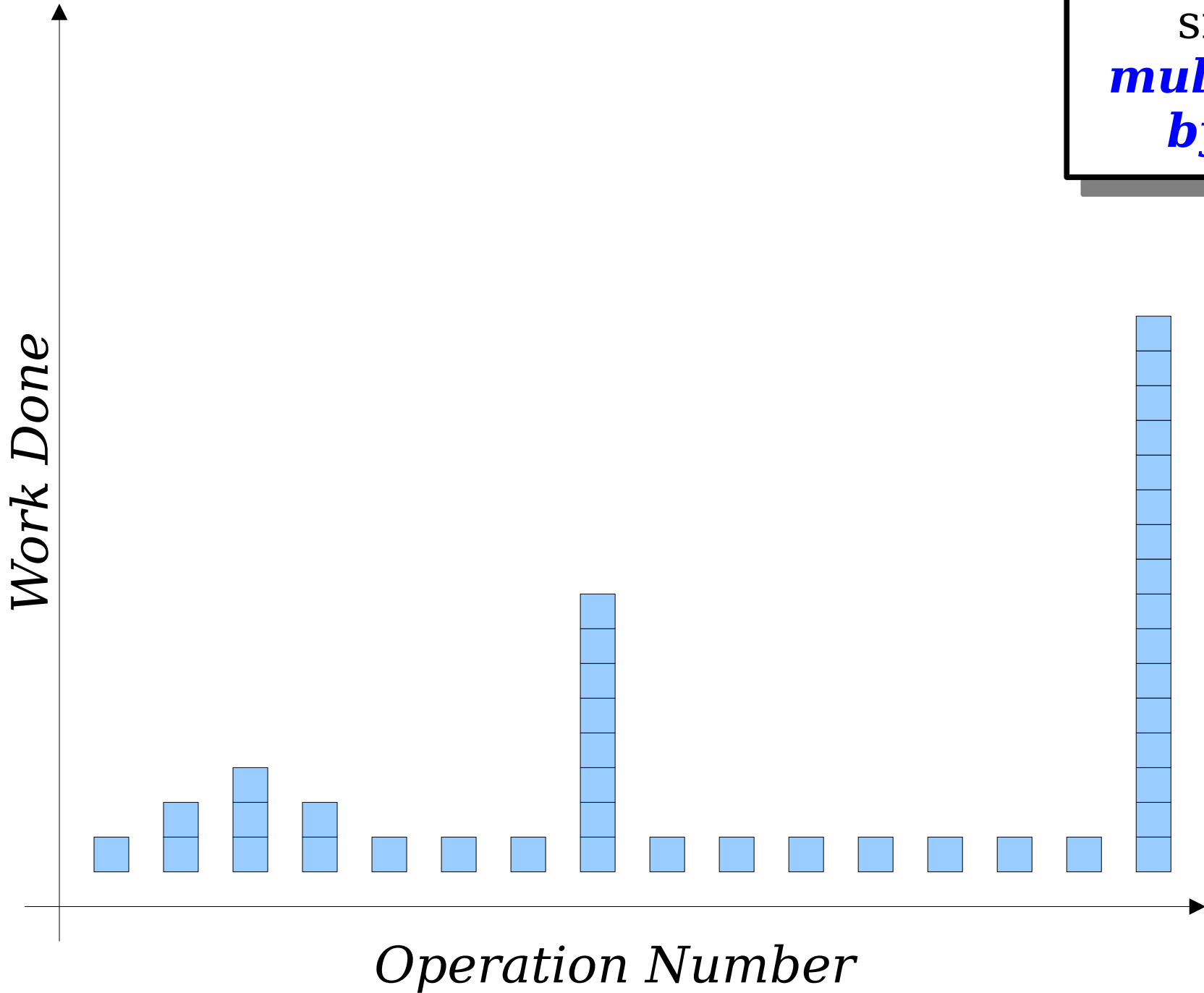
Increase array size by *multiplying by two.*



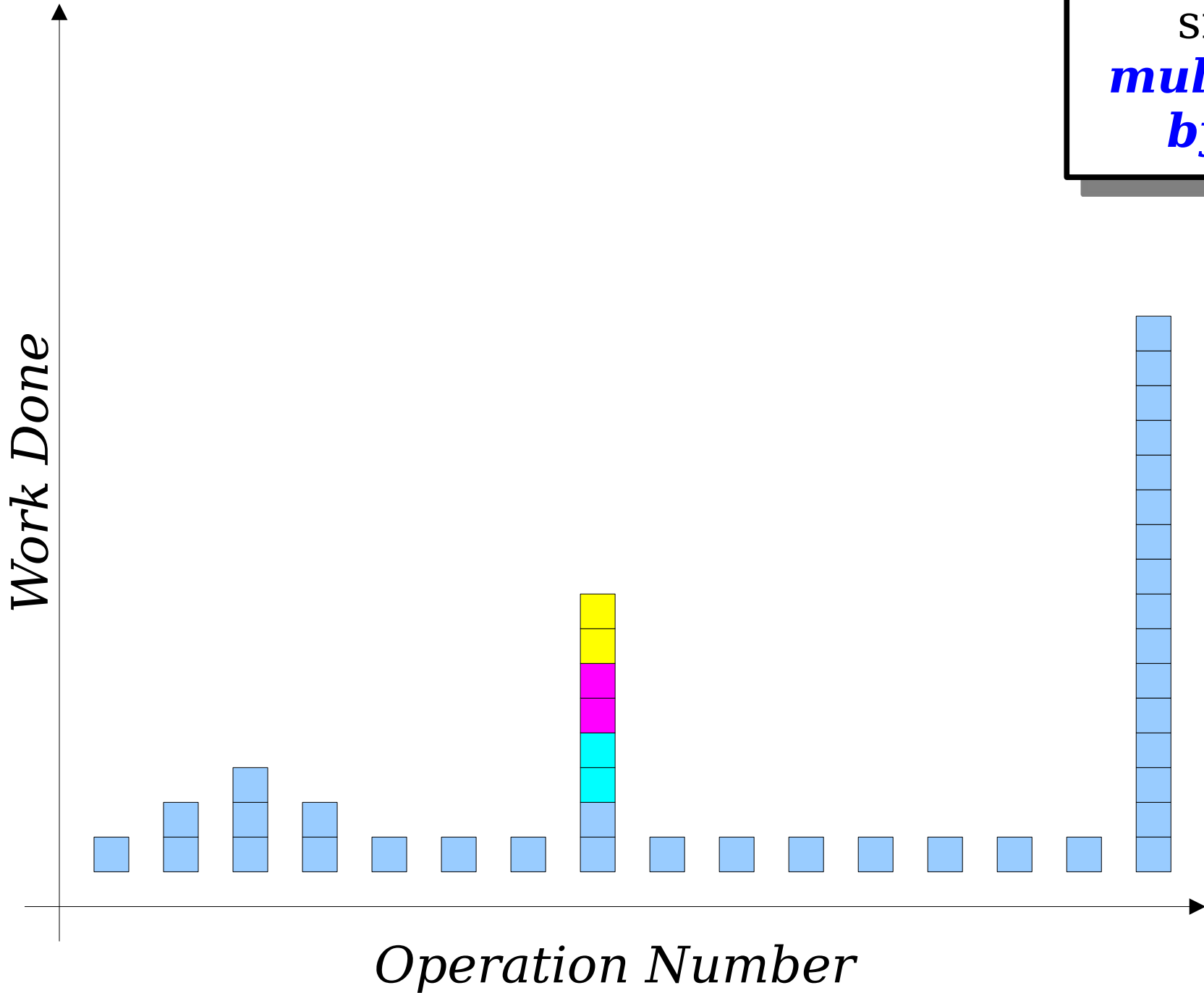
Increase array size by
size by
***multiplying
by two.***



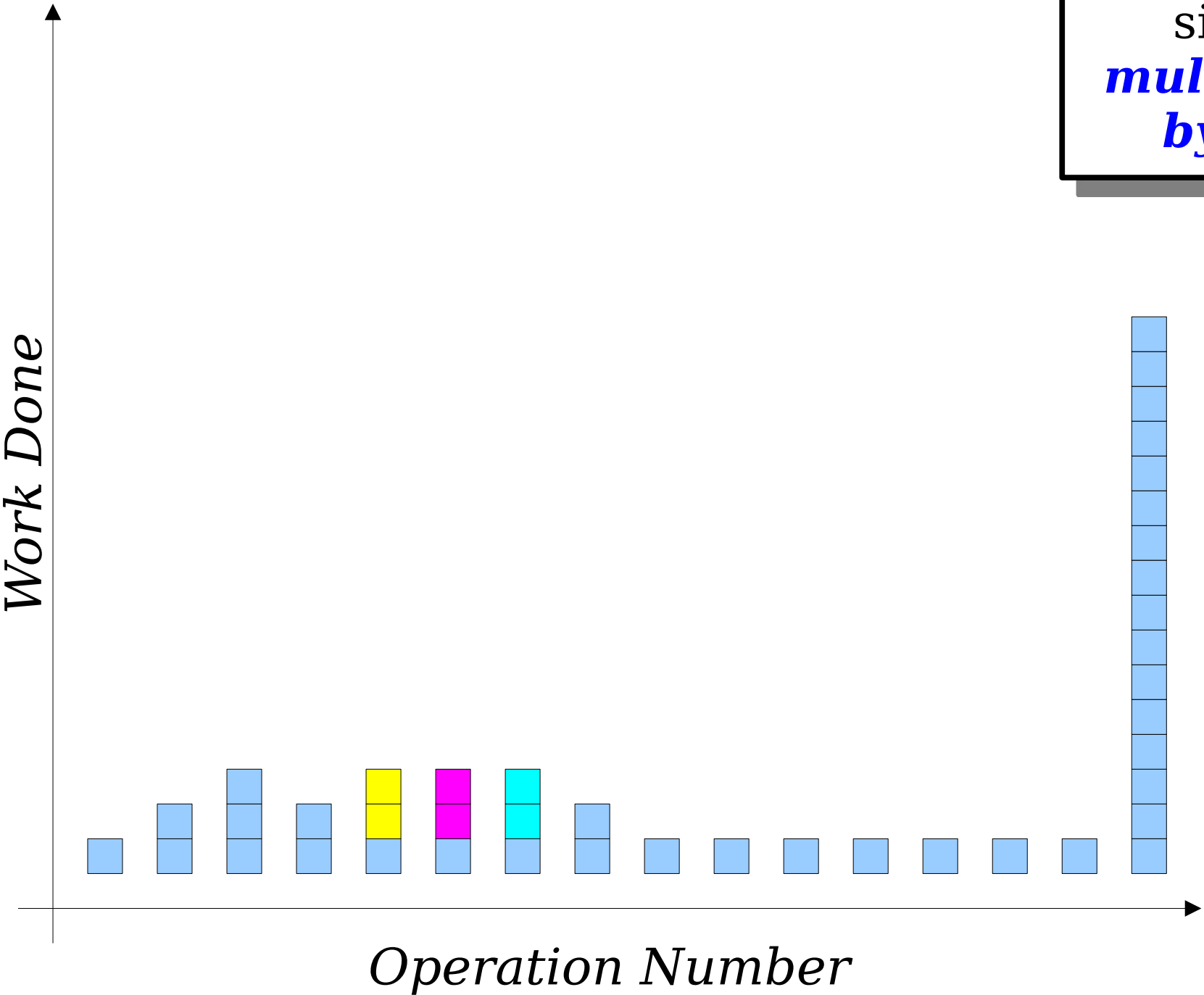
Increase array size by
size by
multiplying
by two.



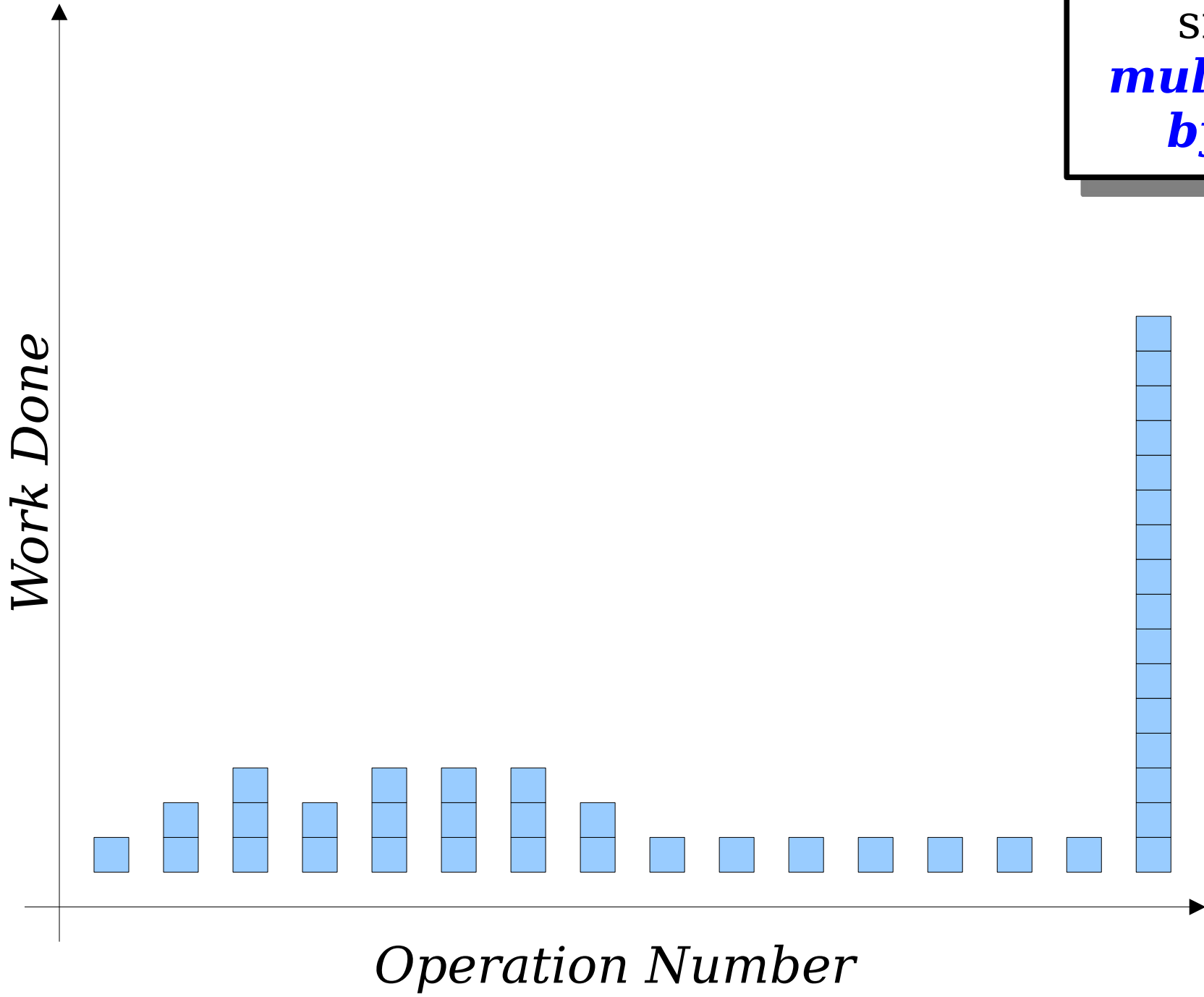
Increase array size by
size by
***multiplying
by two.***



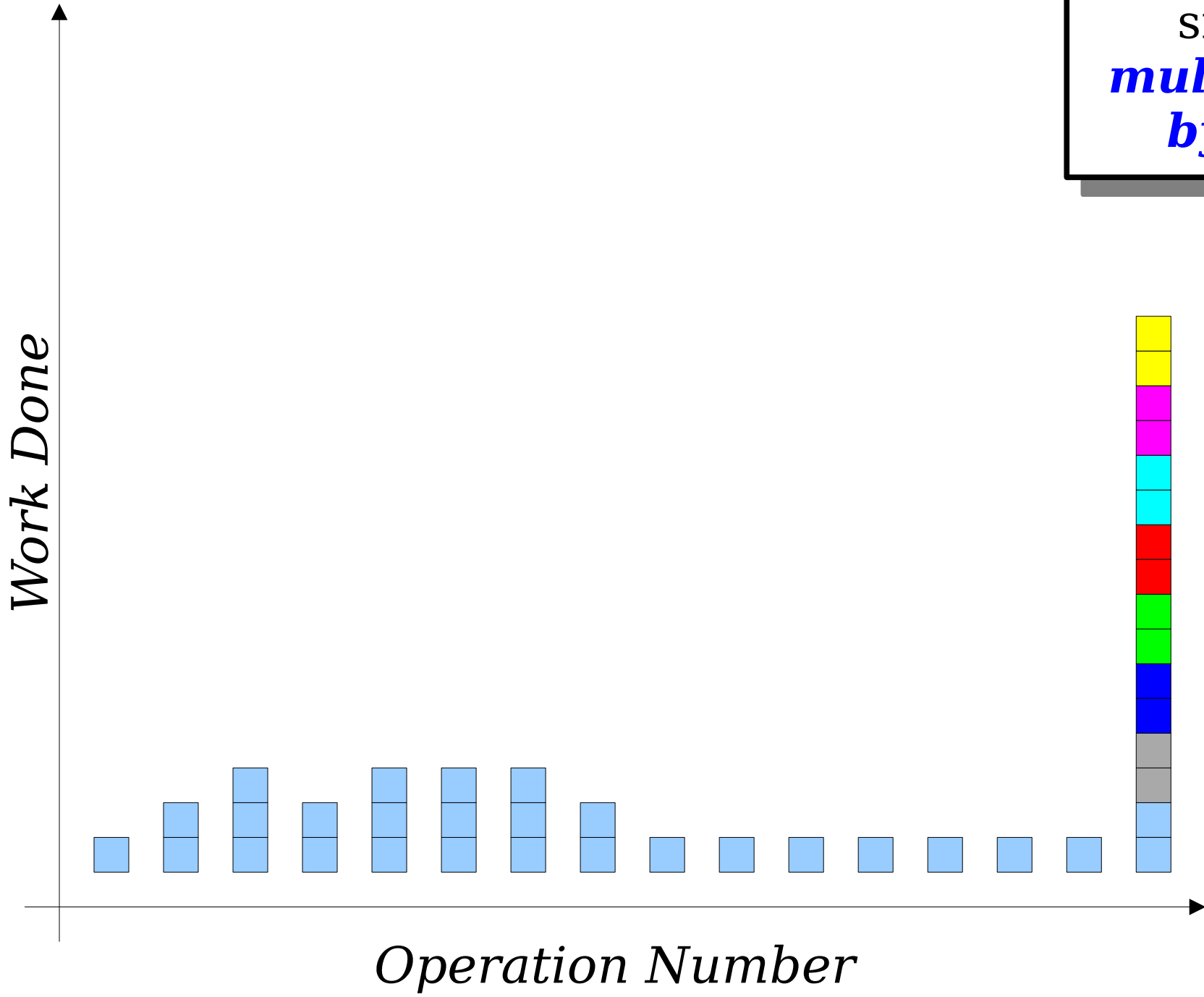
Increase array size by *multiplying by two*.



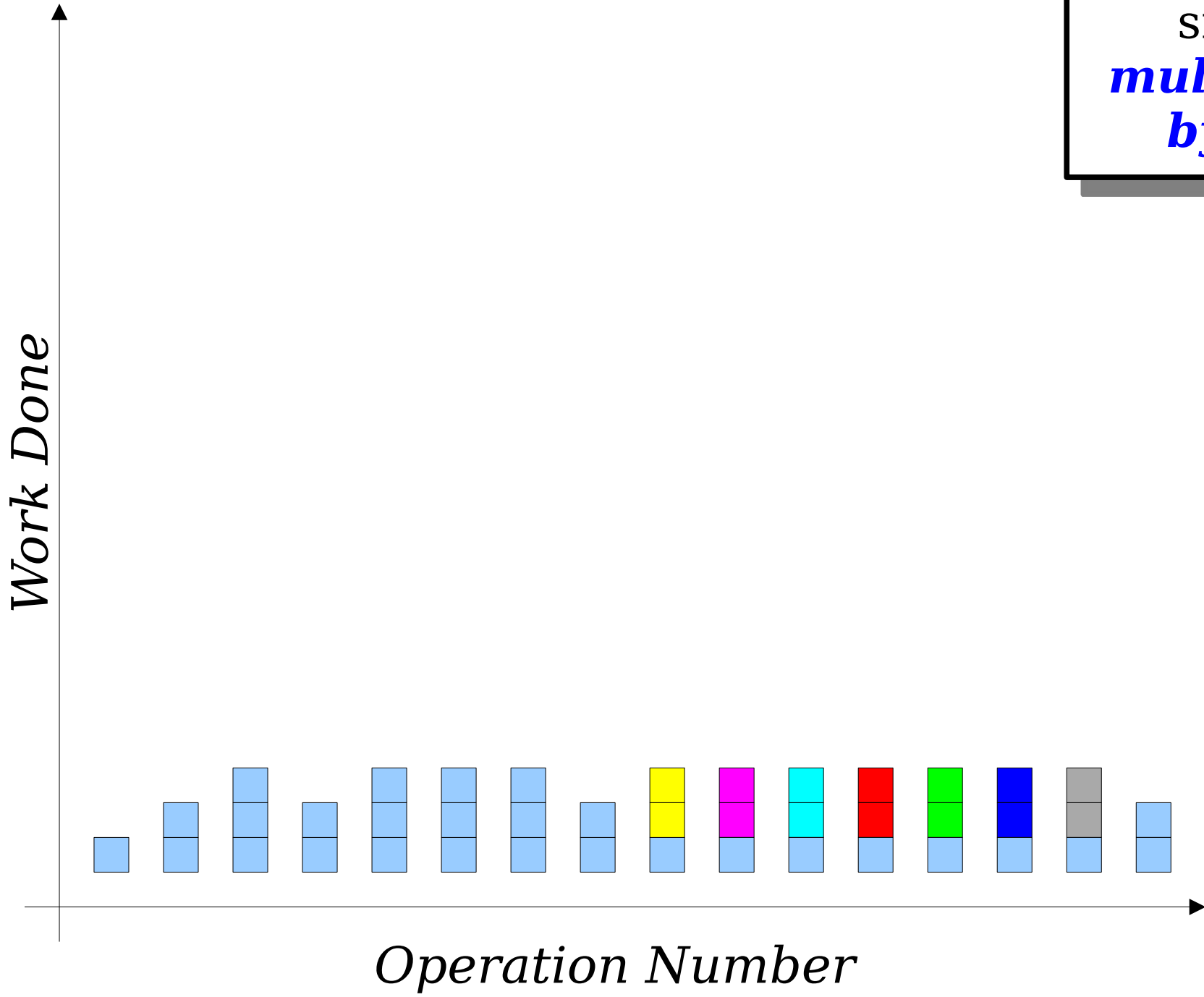
Increase array size by
size by
multiplying
by two.



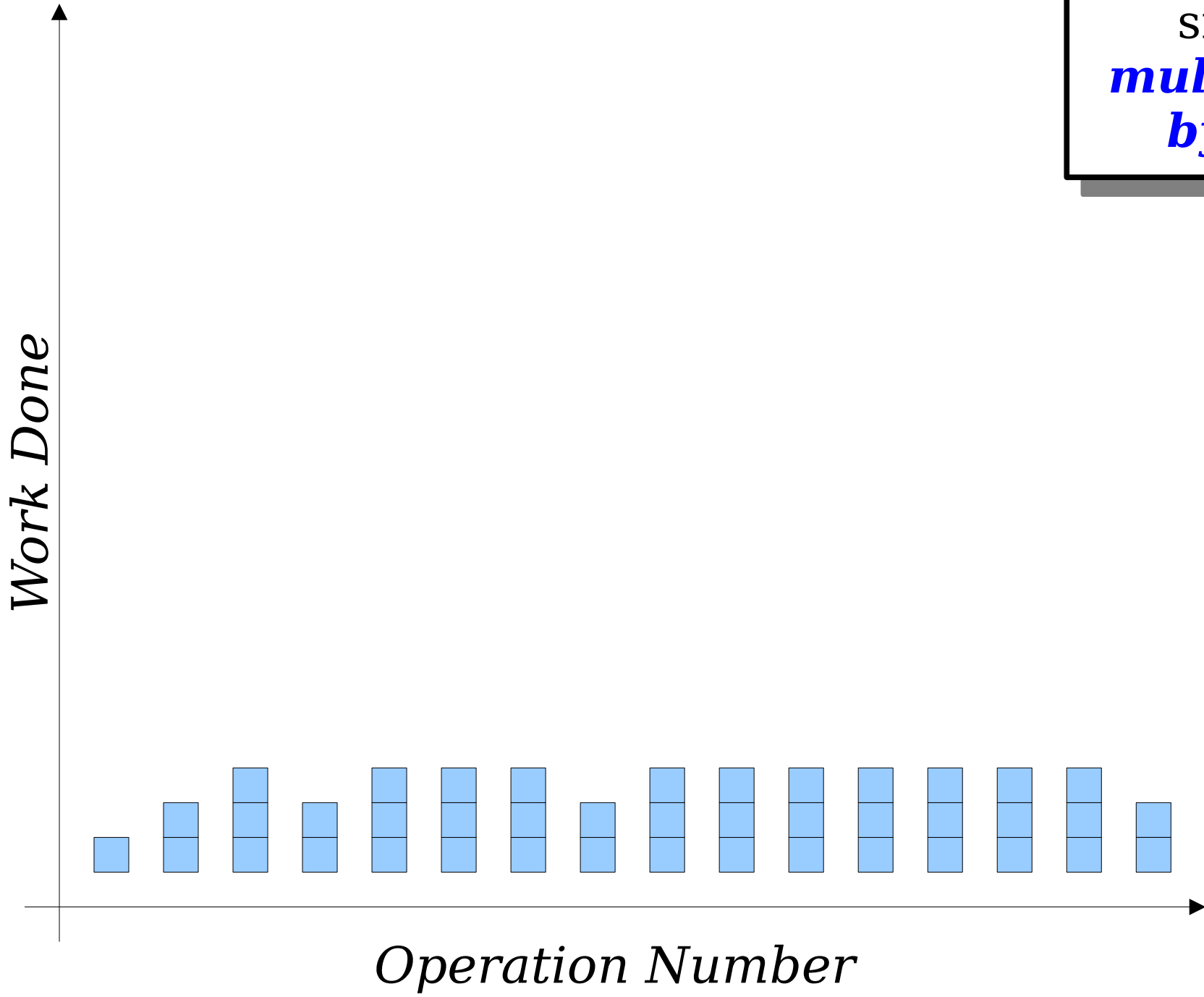
Increase array size by *multiplying by two*.



Increase array size by *multiplying by two.*

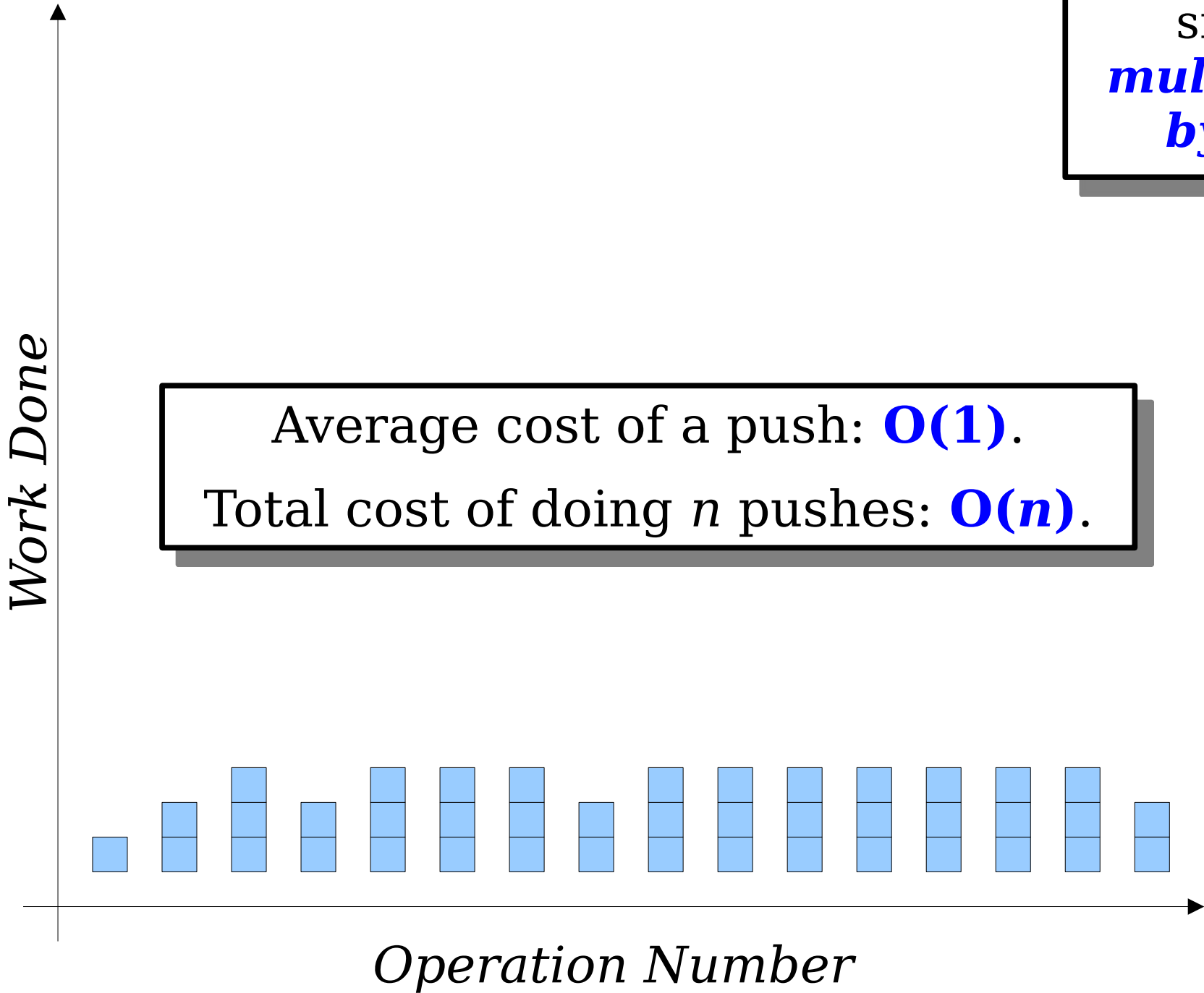


Increase array size by *multiplying by two.*



Increase array size by *multiplying by two*.

Average cost of a push: **$O(1)$** .
Total cost of doing n pushes: **$O(n)$** .



Amortized Analysis

- The analysis we have just done is called an *amortized analysis*.
- We reason about the total work done by allowing ourselves to backcharge work to previous operations, then look at the “average” amount of work done per operation.
- In an amortized sense, our implementation of the stack is extremely fast!
- This is one of the most common approaches to implementing Stack (and Vector, for that matter).

Summary for Today

- We can make our stack grow by creating new arrays any time we run out of space.
- Growing that array by one extra slot or two extra slots uses little memory, but makes pushes expensive (average cost $O(n)$).
- Doubling the size of the array when we run out of space uses more memory, but makes pushes cheap (amortized cost $O(1)$).
- In practice, it's worth paying this slight space cost for a marked improvement in runtime.

Your Action Items

- ***Read Chapter 11 and Chapter 12.1***
 - There's a lot of useful information there about dynamic memory allocation and class design.
- ***Start Assignment 5.***
 - Aim to complete Debugging Warmups tonight and String Simulation by Monday at the start of lecture.
 - Ask for help if you need it! That's what we're here for.

Next Time

- ***No Class Monday***
- ***Then, When We Get Back...***
 - ***Hash Functions***
 - A magical and wonderful gift from the world of mathematics.
 - ***Hash Tables***
 - How do we implement Map and Set?